Conventional Monetary Policies for Unconventional Times: Tracking Monetary Policy Bounds Using Microheterogeneity

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1 Introduction

The modern economy is not only globalised, but often technically complex. Firms buy bespoke inputs from suppliers and sell their products onward as a customised input to another firm. The wait time between placing an order and receiving the first shipment can last several years. For example, in the automotive industry, a lead time of five years for microchips is not uncommon[1]. This means that an input to one firm can not be directly used by another firm; nor is it straightforward for a firm to substitute one supplier for another in the short-run. "Relationship specificity" and the staggered contracting it induces have been well-explored in contract theory, but have gained attention in macroeconomics only recently. Another issue we want to bring to attention is the role of asymmetric information and limited liability in these supplier-buyer relationships.

We explore this combined friction in the context of a supply disaster, where supplier firms throughout the entire economy are hit by productivity shocks. Some firms are hit so hard that the current contract is not only a net loss, but the damage is so severe that they will become insolvent and exit the market without upholding their contractual obligations. Therefore, firms can invoke a "Force Majeure"-clause, triggering a mitigation process with their buyer, where renegotiation is a potential outcome. This provides a means for self-financing within the business relationship, but it opens the door for false claims if idiosyncratic productivity shocks are not publicly observable.

We show that monetary policy reacting to a supply disaster to stabilize prices through tightening will reduce the volume of renegotiation. This is because higher supplier value will be shown to act as a mitigating factor for informational frictions in Force Majeure contracts. As monetary policy tightens, firm value decreases, renegotiation shrinks, and aggregate supply decreases as supplier default amplifies, up to the point where hiking the nominal interest rate increases inflation rather than decreasing it. This is the higher reversal rate we are interested in, in the same spirit as the lower reversal rate for expansionary monetary policy in Abadi et al. (2023).

[1] Chips are designed to specifications that are jointly determined during the engineering cycle.

We will start by presenting in section 2 our micro-foundation for supplier-buyer contracting in anticipation of Force Majeure events. We will first frame non-parametrically our general results, followed by a parametric illustration in partial equilibrium. In section 3, we take our micro-model to the general equilibrium in a representative agent economy. We derive conditions for which reversal arises in this framework. In section 4, we contrast the conclusions of section 3 with the case of a simple heterogeneous agents economy, where households are split between pure hand-to-mouth workers and pure investors.

2 A micro-model of supplier-client relationship

This paper is interested in the response of supply chains to sudden large disruptions to production, such as the COVID-19 pandemic and the War in Ukraine. At the micro-level, the relevant pricing frictions to the amplification of these shocks are staggered contracts between suppliers and clients and the accompanying terms for renegotiations when shocks hit. Different from the retail price change probability found in the majority of the macro-literature (e.g. Calvo pricing), renegotiation terms are endogenous outcomes that we micro-found in this section. As for why supply contracts are staggered in the first place, this will remain out of the scope of our analysis, and we will simply take it as given. The pervasiveness of these contracts in the data[2] might be driven by the cost of establishing supplier-buyer relationships (MacKay (2022)), by the customization of intermediary products for complex, technology-intensive, final products, and by the insurance value of pre-contracting (in contexts of supply shortages a la Bolton and Whinston (1993)).

The main legal framework for renegotiation following production disruptions is Force Majeure clauses, also known as Act of God clauses. While what exact form renegotiation takes differs across contracts, Force Majeure clauses share a common outline: they entail suppliers to claim a shock and ask for renegotiation, while clients reserve themself the right to agree on renegotiation, refuse it or/and switch to a different supplier. Clients ask for evidence of the shock and can generally fact-check this evidence using risk management intermediaries that assess it at low cost (see this account by the Supply Chain Management Review on the subject). The fact that evidence is asked by clients points to asymmetries of information about the suppliers' real productivity.

The model detailed below will capture how asymmetry of information and limited liability from suppliers will shape Force Majeure Clauses. For tractability, we derive the optimal Force Majeure terms in a stylized framework with: a simple two-tier

[2]"Most freight transactions — between 80 and 90 percent, on average — are executed through contracts", as opposed to spot transactions, according to DAT Freight Analytics, a major US-based provider of transportation information: see reference here.

supply chain with only two suppliers and one buyer, in a static model, with renegotiation taking the form of a transfer from the client to the Force Majeure claimant. We use a binary specification for the shock (firms are shocked with a known scale or non-shocked). We will assume away strategic interactions in Force Majeure claims from suppliers, assuming only one of the two suppliers is potentially shocked.

All results for the micro-model will be presented non-parametrically until 2.4, when we discuss mechanisms.

2.1 Framework

One final good producer, also referred to as the client, deals with two intermediate good suppliers indexed by $i = 1, 2$ and assumed to be symmetrical ex-ante. The client and each of its suppliers meet independently before production takes place and agree on quantity and price bundles $\{(y_i, p_i)\}_{i=1,2}$. To fix things, assume the bargaining power at this stage fully goes to the suppliers, who maximize their profits, taking the client's demand schedule, their competitor's price, and their expected marginal cost as given. The final good producer is a representative competitive firm facing a downward-sloping demand curve.

Assume the economy is prone to no uncertainty other than unforeseen, idiosyncratic, productivity shock to the suppliers, referred to as a Force Majeure. This shock was not insured nor contracted on specifically. Firms recognize that a disaster can suddenly happen, but the forms that this disaster can take (pandemic, natural catastrophe, war...) span too large of a range for firms to contract on its specific features, to invest in resilience sufficiently to eradicate all the possible forms of the disaster, or to be insured against all. Instead, the client signs with its suppliers a Force Majeure clause that defines what ensues from a supplier undergoing idiosyncratically the Force Majeure shock. So in a nutshell, this clause is intended to deal with the residual risk that was not insured nor prevented with investments in resilience.

When a shock hits a supplier's productivity sufficiently intensely to make it non-solvent under the pre-contracted quantity and price, it is likely in the client's best interest to allow for renegotiation, avoiding that the supplier defaults without providing any quantity to the client at all. We assume that in the process of renegotiation, bargaining power is passed down to the client, as by definition the shocked supplier has no outside option to a contract he is liable to by law unless he files for bankruptcy. We model renegotiation as a fixed transfer T from the client to the supplier, keeping pre-contracted quantity and price constant. By virtue of the client having all bargaining power, this transfer is set to the minimum amount that keeps the supplier able to operate. This mode of renegotiation is chosen for ease of derivations. Surely, accepting a lower quantity that makes the supplier just solvent could dominate a transfer achieving solvency under constant quantity, assuming strictly decreasing returns to scale for the supplier. But apart from the fact that it is less tractable, there are reasons why this form of renegotiation might not be chosen by the client. For instance, if the client himself is an intermediate good producer and is also liable through pre-contracted quantities to the downstream production tier. Or if the client is a highly scrutinized big firm that finds reputational value in keeping its volume constant.

Now, crucially, idiosyncratic shocks on a supplier's productivity are assumed to be observable only by the supplier itself. This informational friction makes non-shocked suppliers incentivized to falsely declare undergoing a shock and get the transfer T. The client though is assumed to possess an audit technology that reveals the claimant's true type with probability k; with probability $1-k$, the audit is fully uninformative. We assume this audit is costless (in line with the evidence cited above of the low cost of supply chain information providers' services).

When the audit reveals a non-shocked firm lied to claim the transfer T, we assume it is sued by the client for its assets, losing them all. We assume for simplicity that these assets exactly cover for the client's legal fees, and we assume the supplier then exits without producing. So in terms of outcome for the client, this node of the mechanism is equivalent to the one that has the shocked supplier filing for bankruptcy and defaulting. When the audit reveals a claimant was truly shocked, the client grants the transfer T to the truthful supplier.

Now, the case that remains is when the audit fails to reveal the supplier's type, a case we will refer to as the "indeterminacy" case. We will model the client's response in indeterminacy as a probability a of granting renegotiation, to which the client commits on. We assume that the client has commitment power through intermediaries to which it can delegate the application of the contract. a is an object representing the client's leniency, or equivalently how he is willing to balance between type I and type II errors in granting renegotiation under indeterminacy. The value of a does not have to be committed to in advance and be written in the contract: just like T, a can be determined after having observed the realization of the scale of the shock and other relevant parameters, and be committed to only at that stage.

If the firm does not claim a shock, it remains liable under the pre-contracted quantity and price. The following graph synthesizes the mechanism and the payoffs that follow to suppliers at each node, relative to not claiming Force Majeure :

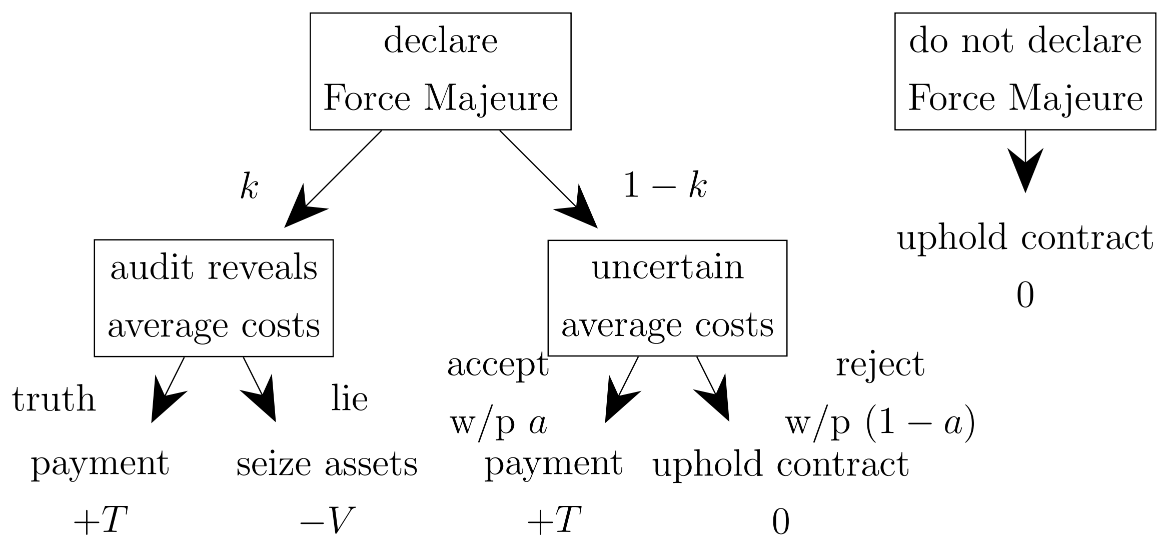

Figure 1: a is the endogenous object committed on by the buyer.

To fully solve for the optimal Force Majeure contract, we now turn to the determination of the optimal a.

2.2 Solving the model

To fix things, assume:

- only one of the two suppliers can be affected by the shock at a time. Which supplier was potentially shocked is common knowledge. Without loss of generality and to fix things, assume it is supplier $i = 1$ that was potentially shocked.
- at the time of the shock, the scale of the shock and the probability that supplier 1 was indeed shocked are common knowledge
- the shock is high enough that supplier 1 is insolvent if it was really shocked.

Denote: $V_{NS} > 0$ and $V_S < 0$ the supplier 1's asset value conditional on being non-shocked and shocked respectively.

Denote $T = |V_S| = -V_S$ the contractual transfer needed to keep the shocked firm operating: with this transfer, the shocked firm value is exactly 0, and it can keep operating. Note this is withholding any borrowing constraint. Denote Π^C the profit of the final good firm, and $\mathbb{E}(\Pi^C \mid a)$ the expectation of this profit conditional on the value of a. This expectation is thus taken over the probability of firm 1 to be truly shocked and given the best response (claim or not claim) of each type to the value a. Denote as $\Pi^C(\mathbf{R}), \Pi^C(\bar{\mathbf{R}}), \Pi^C(\mathbf{E})$ the profits of the final good firm respectively in case of renegotiation, production as usual (a case that arises when the firm is non-shocked and decides not claiming renegotiation), and in case of firm exit (either because a lying firm was sued or because a shocked firm did not get renegotiation and filed for bankruptcy). We will now derive the optimal contract, under three assumptions.

Assumption 1. $\Pi^C(\mathbf{R}) > \Pi^C(\mathbf{E})$

Assumption 2. *If* $1 > a^{Sep}$*:* $\mathbb{E}(\Pi^C \mid a = a^{Sep}) > \mathbb{E}(\Pi^C \mid a = 1)$ *where* a^{Sep} *is the value of a making the non-shocked firm indifferent between claiming and not claiming.*

Assumption 3. *Assume that when indifferent between claiming and not claiming, the non-shocked firm decides not claiming.*

Comments:

- Note that Assumption 1 and Assumption 3 imply that $\mathbb{E}(\Pi^C \mid a \leq a^{Sep})$ and $\mathbb{E}(\Pi^C \mid a > a^{Sep})$ being respectively strictly increasing functions of a.

- Let us flesh out what Assumption 2 entails. Denote ω the probability of firm 1 being non-shocked. Then:

$$\begin{aligned}
&\mathbb{E}(\Pi^C \mid a = a^{Sep}) \geq \mathbb{E}(\Pi^C \mid a = 1) \Leftrightarrow \omega\Pi^C(\bar{\mathbf{R}}) + (1-k)(1-\omega)(1-a^{Sep})\Pi^C(\mathbf{E}) \\
&+ (1-\omega)((1-k)a^{Sep} + k)\Pi^C(\mathbf{R}) \geq \omega k\Pi^C(\mathbf{E}) + (1-\omega k)\Pi^C(\mathbf{R}) \\
&\Leftrightarrow k\omega(\Pi^C(\bar{\mathbf{R}}) - \Pi^C(\mathbf{E})) + (1-k)\omega(\Pi^C(\bar{\mathbf{R}}) - \Pi^C(\mathbf{R})) \geq (1-\omega)(1-k)(1-a^{Sep})(\Pi^C(\mathbf{R}) - \Pi^C(\mathbf{E})) \\
&\Leftrightarrow T \geq (1-\omega)(1-k)(1-a^{Sep} - \omega(1-k))(\Pi^C(\mathbf{R}) - \Pi^C(\mathbf{E}))
\end{aligned}$$

  where the second to last line lends itself to easier interpretations: in the LHS is the sum of what the client wins from non-shocked firms being deterred from claiming compared to the counterfactual where they would have been caught lying and sued, and of what the client wins from non-shocked firms being deterred from claiming compared to the counterfactual where they would have been granted the transfer because of audit resulting in indeterminacy. In the RHS is the loss from shocked firm exiting because of $a^{Sep} < 1$.

- Note that we don't have constraints on ensuring that the shocked firms claim renegotiation, as their outside option of not claiming leads necessarily to default. So whenever $0 < k$, claiming strictly dominates for shocked firms.

Theorem. *Under the above assumptions, and with $k < 1$, the optimal contract sets:*

$$a^* = \min\{a^{Sep}, 1\} = \min\left\{\frac{k}{1-k}\frac{V_{NS}}{T}, 1\right\}$$

so that the client separates between shocked firms and non-shocked firms, non-shocked firms being deterred from claiming Force Majeure.

Proof. By Assumption 1 and Assumption 3, $\mathbb{E}(\Pi^C \mid a \leq a^{Sep})$ and $\mathbb{E}(\Pi^C \mid a > a^{Sep})$ are respectively strictly increasing in a. So their maximums are attained at the right bound of each of their respective interval of definition. By Assumption 2, if $1 < a^{Sep}$ this maximum for the left interval is strictly higher than the one on the right interval. Then, the expression for a^{Sep} is derived from non-shocked supplier indifference condition:

$$\begin{aligned} V_{NS} &= k \cdot 0 + (1-k)a^{Sep}(V_{NS} + T) + (1-k)(1-a^{Sep})V_{NS} \\ \Leftrightarrow V_{NS}(1-(1-k)) &= (1-k)a^{Sep}T \\ a^{Sep} &= \frac{k}{1-k}\frac{V_{NS}}{T} \end{aligned}$$

If $a^{Sep} \geq 1$, then from $\mathbb{E}(\Pi^C \mid a \leq a^{Sep})$ being strictly increasing, the optimal contract sets $a = 1$. $\square$

2.3 Mechanisms

We turn to the interpretation of the optimal contract. Naturally, the efficiency of the audit unambiguously shifts up the probability of renegotiation under uncertainty, as it facilitates deterrence of the non-shocked firms. Besides that, the most striking feature of the optimal contract is the role played by firm valuation. Higher firm valuation has a double effect:

- conditional on them being insolvent, that shocked firm's value is higher leads to lower transfers to keep them operating, so lower return on lying for non-shocked firms
- higher firm value also means higher skin in the game for non-shocked firms that are threatened to lose all of their value when caught lying

Higher firm valuations can even erase inefficiencies altogether, leading to $a^{Sep} \geq 1$ and renegotiation being always granted. Finally, higher firm valuation also makes assumption 1. more likely to hold, as it increases $\Pi^C(\mathbf{R})$ through the decrease of

T. Assumption 2. might not hold with T decreasing, but it is rightly because full renegotiation is then preferred over partial renegotiation.

Now, this is where the potential for general equilibrium amplification can arise. It is a well-established empirical fact that the stock market's valuations react negatively to a tightening of rates (Bernanke and Kuttner (2005)). So in conducting tightening monetary policy in times of Force Majeure, it is sensible to ask whether the monetary authority isn't amplifying firm exits by shifting the optimal amount of renegotiation down, and even proving counterproductive by eventually hiking prices. While the literature on monetary policy during supply shocks focuses on its implication for the intensive margin of supply (the level of investment, in particular), this paper will generate results only on the extensive margin, deliberately shutting down channels for intensive margin effects.

2.4 A parametric example in partial equilibrium

In this part, we parameterize the micro-model and derive in partial equilibrium the impact of higher rates on firm exit. This is just for the sake of illustrating the above model, as we are ultimately interested in moving on to the general equilibrium. Assume the final good is produced following a CES-aggregator:

$$Y(\{y(s)\}_s) = \left[\int y(s)^{\frac{\epsilon-1}{\epsilon}} \, ds\right]^{\frac{\epsilon}{\epsilon-1}}$$

where $y(s)$ is the amount of type s-input used, and ϵ is the constant elasticity of substitution.

Cost minimisation yields the demand for intermediate input goods:

$$\min_{y(s)} \int p(s)y(s) \, ds \quad s.t. \quad Y(\{y(s)\}_s) \geq Y$$

First in relative terms between varieties s and t:

$$\implies [y(s)]: \; p(w) = \mu \Big[\int y(s)^{\frac{\epsilon-1}{\epsilon}} \; ds \Big]^{\frac{\epsilon}{\epsilon-1}-1} \; y(w)^{-\frac{1}{\epsilon}}$$
$$\implies \frac{p(s)}{p(t)} = \Big(\frac{y(s)}{y(t)}\Big)^{-\frac{1}{\epsilon}}$$
$$\implies y(s) = \Big(\frac{p(s)}{p(t)}\Big)^{-\epsilon} y(t)$$

And then in terms of level of final output Y:

$$Y = \Big[\int \Big(\underbrace{\Big(\frac{p(s)}{p(t)}\Big)^{-\epsilon} y(t)}_{y(s)} \Big)^{\frac{\epsilon-1}{\epsilon}} \; ds \Big]^{\frac{\epsilon}{\epsilon-1}}$$
$$= \Big[\int p(s)^{1-\epsilon} \; ds \Big]^{\frac{\epsilon}{\epsilon-1}} \; p(t)^{\epsilon} y(t)$$
$$\implies y(t) = p(t)^{-\epsilon} \Big(\underbrace{\Big[\int p(s)^{1-\epsilon} \; ds \Big]^{\frac{1}{1-\epsilon}}}_{=:P} \Big)^{\epsilon} Y$$
$$\implies y_s(p_s) = \Big(\frac{p_s}{P}\Big)^{-\epsilon} Y$$

Changing notation for variety indexation in the last line, this is the demand curve for intermediate inputs of variety $s \in [0,1]$, which the intermediate good supplier takes as given. Here, we have defined the price index P over input goods as $\Big[\int p(s)^{1-\epsilon} \; ds \Big]^{\frac{1}{1-\epsilon}}$.

The supplier s takes wages w as given in each period, and produces with a CRS production function using only labour n_s as input:

$$y_s = z_s n_s$$

with z_s being the firm-specific productivity. During normal times, $z_s = 1 \; \forall s$. Therefore, all suppliers face the same, constant marginal costs for producing y_s of $c = c_s = \frac{w}{z_s}$. These are also the average costs.

Facing the CES factor demand curve of the final good firm, the supplier of variety s sets prices statically, under constant marginal costs:

$$
\begin{aligned}
&\max_{p_s} \quad p_s \cdot y_s(p_s) - c \cdot y_s(p_s) \\
= &\max_{p_s} \quad p_s \cdot \left(\frac{p_s}{P}\right)^{-\epsilon} Y - c \cdot \left(\frac{p_s}{P}\right)^{-\epsilon} Y \\
&[p_s] : (1-\epsilon)\left(\frac{p_s}{P}\right)^{-\epsilon} Y - c(-\epsilon)\left(\frac{p_s}{P}\right)^{-\epsilon} \frac{1}{p_s} Y = 0 \\
&\implies (1-\epsilon) = c(-\epsilon)\frac{1}{p_s} \\
&\implies p_s^* = \frac{\epsilon}{\epsilon - 1} c
\end{aligned}
$$

Intermediate input suppliers conventionally charge a proportional markup over constant marginal costs. Firm profit in any "normal times" outside the time of the shock are given by the constant expression:

$$
\begin{aligned}
\Pi_s &:= (p_s^* - c) \cdot y_s(p_s^*) \\
&= (p_s^* - c) \cdot \overbrace{\left(\frac{p_s^*}{P}\right)^{-\epsilon} Y} \\
&= \left(\frac{\epsilon}{\epsilon - 1} c - \frac{\epsilon - 1}{\epsilon - 1} c\right)\left(\frac{\frac{\epsilon}{\epsilon-1} c}{P}\right)^{\epsilon} Y \\
&= \left(\frac{\epsilon}{P}\right)^{-\epsilon}\left(\frac{c}{\epsilon - 1}\right)^{1-\epsilon} Y \\
&\equiv \Pi(Y)
\end{aligned}
$$

Assume the economy is at a steady state with no inflation, such that the nominal rate equals the real rate. By no-arbitrage with bonds emitted by the monetary authority, real equity value of intermediate good firms equals the cash flow of the firm discounted by $\frac{1}{1+r}$:

$$\begin{aligned} V_s(r) = \sum_{t=0}^{\infty} \beta^t \Pi_s \, ds &= \frac{\Pi_s}{1-\beta} \\ &= \frac{\Pi(Y)}{1-\beta} \\ &\overset{\beta=\frac{1}{1+r}}{=} \Pi(Y)\frac{1+r}{r} \end{aligned}$$

With $\beta = \frac{1}{1+r}$ because the economy is in a steady state.

Let us now derive the condition for shocked firms to be critically affected by the shock, becoming insolvent. The idiosyncratic supplier firm productivity falls to $z^{dis} < 1$ in $t = 0$. For convenience in analytical expressions, we parametrize the productivity fall using another parameter $\kappa_s \geq 0$:

$$z^{dis} = \underbrace{\frac{1}{1+\frac{\kappa_s}{\epsilon-1}}}_{\leq 1} \cdot \underbrace{\ z\ }_{=1} \leq 1$$

We'll assume the wage is fixed at the same value throughout periods. Flow profit during the disaster is:

$$\begin{aligned} \underline{\Pi}_s &= \left(p_s - c_s^{dis}\right) \cdot y(p_s^*) \\ &= \left(\frac{\epsilon}{\epsilon-1}c - \big(1 + \frac{\kappa_s}{\epsilon-1}\big)c\right) \cdot y(p_s^*) \\ &= (1-\kappa_s)\frac{c}{\epsilon-1} \cdot \left(\frac{p_s^*}{P}\right)^{-\epsilon} Y \\ &= (1-\kappa_s)\left(\frac{\epsilon}{P}\right)^{-\epsilon}\left(\frac{c}{\epsilon-1}\right)^{1-\epsilon} Y \\ &= (1-\kappa_s)\Pi(Y) \end{aligned}$$

κ_s is the degree of production impairment, with $\kappa_s = 0$ nesting normal times and $\kappa_s > 1$ inducing a net loss on the current project. Comparing disaster times flow profits with those of normal times, they are marked down by a multiplicative term $(1 - \kappa_s)$. Assume from now for exposition that the future is just composed of one period after the shock, so the value of the firm is the sum of today's profits and tomorrow's profits discounted with the real rate. A threshold rule on κ_s characterise the exit decision for firm s as a threshold rule on κ_s:

$$\begin{aligned}
\text{Exit} &\iff \underline{\Pi_s} + \frac{1}{1+r} V_s < 0 \\
&\iff (1-\kappa_s)\Pi(Y) + \frac{1}{1+r}\Pi(Y) < 0 \\
&\iff (1-\kappa_s) + \frac{1}{1+r} < 0 \\
&\iff \kappa_s > \frac{2+r}{1+r}
\end{aligned}$$

with the third line following from $\Pi(Y) > 0$. So we know that any firm for which the supply disaster causes $\kappa_s > \frac{2+r}{1+r}$ would exit if denied renegotiation. The client would have to compensate by a transfer of $T = \big(\kappa_s - \frac{2+r}{1+r}\big)\Pi(Y)$. Also, we have:

$$\begin{aligned}
V_{NS} &= \underline{\Pi_s} + \frac{1}{1+r} V_s \\
&= \Pi(Y) + \frac{1}{1+r}\Pi(Y) \\
&= (\frac{2+r}{1+r})\Pi(Y)
\end{aligned}$$

Now we plug in these expressions in the characterisation for the contract object a:

$$a^* = \min\left\{\frac{k}{1-k}\underbrace{\frac{(\frac{2+r}{1+r})}{(\kappa_s - \frac{2+r}{1+r})}}_{>0}, 1\right\}$$

The key channel we wish to highlight in our model is the extensive margin of firm operations with respect to the real interest rate. This will imply a trade-off for a central bank that aims to increase rates in order to bring down inflation.

How many firms will default on their contractual obligations if $r \uparrow$? Using that $\frac{\partial}{\partial r}\frac{2+r}{1+r} = -\frac{1}{(1+r)^2}$, for all $a^* < 1$:

$$
\begin{aligned}
\frac{\partial}{\partial r}a^*(r) &= \frac{\partial}{\partial r}\frac{k}{1-k}\frac{(\frac{2+r}{1+r})}{(\kappa_s - \frac{2+r}{1+r})} \\
&= \frac{k}{1-k}\Big(\frac{1}{\kappa_s - \frac{2+r}{1+r}}(-\frac{1}{(1+r)^2}) + (\frac{2+r}{1+r})\frac{(-1)}{(\kappa_s - \frac{2+r}{1+r})^2}(\frac{1}{(1+r)^2})\Big) \\
&= \frac{k}{1-k}\Big(\frac{1}{\kappa_s - \frac{2+r}{1+r}}\Big)\Big(\frac{1}{(1+r)^2}\Big)\underbrace{\Big(-1 - \frac{\frac{2+r}{1+r}}{\kappa_s - \frac{2+r}{1+r}}\Big)}_{\frac{-\kappa_s}{\kappa_s - \frac{2+r}{1+r}}} \\
&= \frac{k}{1-k}\frac{-\kappa_s}{\big(\kappa_s - \frac{2+r}{1+r}\big)^2}\frac{1}{(1+r)^2} < 0
\end{aligned}
$$

The derivative is negative because $\kappa_s > 0$.

In a mass of shocked firms normalized to 1, the total number of shocked firms that are denied renegotiation, and therefore also the fraction of firms that exit the market, is given by

$$(1-k)(1-a^*(r))$$

Overall, for a marginal increase in the real interest rate, firm exits increase by

$$(1-k)(-\frac{\partial}{\partial r}a^*(r)) > 0$$

3 General Equilibrium with a continuum of firms - RANK

In this part, we embed the micro-model for renegotiation claims in the General Equilibrium. As a first benchmark, we focus on an economy with a representative agent. This framework will be modified in the next section, where a simple form of household heterogeneity will be introduced.

3.1 Timing and Agents

Time is discrete and infinite. There is no uncertainty in the economy, except for a Force Majeure event that will hit the economy at a time period fixed without loss of generality at time $t = 0$.

Firms

Similar to the micro-model, we consider a simple two-tier structure, with a representative competitive final good producer engaged in staggered contracting with its suppliers, who are monopolistically producing. As in the above partial equilibrium example, We will assume the final good producer deals with a continuum of suppliers, with the probability of being non-shocked conditionally on being potentially shocked ω now turning into a measure of firm. This is to use the law of large numbers for determining the mass of firms exiting given the Force Majeure clause, hence making the path of the economy deterministic in the aftermath of the shock, and considerably simplifying the analysis. It is worth underlining that taking the micro-model to its population version makes the analysis even more reliant on the assumption of no strategic interactions in claims. In the present framework, contrary to the micro-model framework, non-shocked but potentially shocked firms will cohabit with shocked firms.

Firms and the client are engaged in staggered contracting, deciding at time $t-1$ on quantity and price for "normal times" supply for time t. In addition to these "normal times" bundles, contracts signed at time $t-1$ include a Force Majeure clause. The clause defines what ensues from the supplier claiming a renegotiation, exactly as spelled out in the previous section.

We will impose in the analysis that the assumptions 1. to 3. listed in section 2 are verified. It is then common knowledge that at the time of the shock, the optimal a is given by:

$$a^* = \min \left\{ \frac{k}{1-k} \frac{V_{NS}}{T}, 1 \right\} \tag{1}$$

To keep the micro-model sensible in the dynamic General Equilibrium, assume incumbent firms that exit at the time of the shock will be replaced in the subsequent period by firms that have "normal times" productivity. This essentially keeps the optimal contract we derived in a static setting valid in this DSGE.

For production functions, assume the client aggregates its suppliers production with a CES-aggregator:

$$Y(\{y(s)\}_s) = \left[\int y(s)^{\frac{\epsilon-1}{\epsilon}} \, ds \right]^{\frac{\epsilon}{\epsilon-1}}$$

With $\epsilon > 1$.

Assume the suppliers have constant return-to-scale production function, using only labor to produce. We normalize their normal-times productivity factor to 1:

$$y_s = n_s$$

Households

The economy is populated by a representative household with GHH preferences:

$$U(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{n^{1+\psi}}{1+\psi}$$

The household maximizes the discounted sum of his instantaneous utilities, discounted at the rate β.

Expectations on productivity shocks

Note that above we assumed consumers, the client, and the suppliers had deterministic objective functions, without mention of expectations on the occurrence of the Force Majeure shocks. Here we discuss why this is without loss of generality under light restrictions on the disaster structure. Assume the Force Majeure is an aggregate shock that when hits, takes down to a value $z^{dis} < 1$ the productivity of a measure ω of firms uniformly randomly drawn. Denote S as the process for the state of the world, with $S \in (N, D)$, where N denotes the normal state and D the disaster state. S is assumed to be a Markov process. We assume $\mathbb{P}(S_t = N \mid S_{t-1} = D) = 1$ (the disaster state never lasts more than 1 period).

At a period $t-1$, under the rational expectations hypothesis, the final good firm, that is perfectly competitive, forms its demand schedule of suppliers' differentiated goods based on the expectations of the final good price in the future state of the world, "normal times" or disaster, conditional on its observation of the current state of the world and given the law of S. The client also formulates its Force Majeure clause, and though the client does not have to commit to the value of a^* in advance, suppliers know that under the conditions for theorem 1, the client will implement the separating equilibrium. Assume we restrict the economy to satisfy these conditions.

Crucially, the option of renegotiation will not affect the supplier's pricing choice. The suppliers set simultaneously their proposed price and quantities for the future period t at a level that maximizes their expected profit, given the final good firm price schedule, their expectations for the wage level in each possible state of the world, and their probability to be idiosyncratically affected next period conditional on the disaster state arising. Firms will price weighing the fact that in some states of the world, they will exit if they commit to too high of a quantity produced. However, how this will factor in their decision will not depend directly on the Force Majeure clause, because by definition of T, when shocked, firms will get the same outcome if they are granted renegotiation or if they exit. As for consumers, they will also make saving and working decisions based on the current state of the world and their rational expectations of what will be the prices in the next period, but not based on the existence of the Force Majeure clause. So given this analysis there are no important interactions between the fact that households and firms have priors on the probability of the Disaster, and the Force Majeure contract application. The Force Majeure contract does not skew quantities given the form the renegotiation takes and under the separating equilibrium. For this reason, because we want to focus on what follows the disaster state given the renegotiation form, we will consider households and firms making decisions based on "normal times" lasting forever. And we will consider a path of the economy such that they are right except for period $t = 0$.

Monetary policy

The central bank sets the nominal interest rate i (more on this below).

3.2 The economy in normal times

Under the simplifying assumptions, the economy's equilibrium during "normal times" is described by the following set of equations, describing agents' optimization and market clearing at time t:

Households

$$1 = \beta(1+i)\frac{p_t}{p_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \tag{2}$$

$$\frac{w_t}{p_t} = C_t^{\sigma} n_t^{\psi} \tag{3}$$

$$\frac{1}{1+r_t} = \frac{\pi_{t+1}}{1+i_t} \tag{4}$$

$$\tag{5}$$

With $\pi_t \equiv \frac{p_t}{p_{t-1}}$.

Firms

$$Y_t = y_t^s, \forall s \tag{6}$$

$$p_t^s = \frac{\epsilon}{\epsilon - 1} w_t, \forall s \tag{7}$$

$$p_t = p_t^s, \forall s \tag{8}$$

$$n_t^s = y_t^s, \forall s \tag{9}$$

(s indexes firms).

Labor and Final Good market clearing

$$Y_t = C_t \tag{10}$$

$$n_t = \int n_t^s ds \tag{11}$$

Dynamics

From these equations, we see that the economy will be in a steady state with constant quantities, as:

$$\begin{aligned} \frac{w_t}{p_t} &= \frac{\epsilon - 1}{\epsilon} \\ &= C_t^\sigma n_t^\psi \\ &= Y_t^{\sigma+\psi} \end{aligned}$$

Where we normalized the total mass of firms to 1.

Thus, from the Euler equation, $r = \frac{1}{\beta} - 1$ is pinned down independently of monetary policy, and monetary policy is neutral. This is because, in the absence of the Force Majeure shock arising, the fact that contracts are staggered does not generate any real friction by virtue of rational expectations.

3.3 The economy under the disaster

Now assume a productivity shock suddenly hits at time $t = 0$ a share ω of the firms, under the same conditions of asymmetry of information and limited liability as in the micro-model. These firms' productivity is scaled down by a factor $\Delta_S > 1$. The remaining share $1 - \omega$ of firms is not shocked but is considered as potentially shocked by the final good producer by lack of information. Following the contract terms defined in the previous section, the final good firm accepts renegotiation under audit indeterminacy with the probability a^*:

$$a^* = \frac{k}{1-k}\frac{V_{NS}}{T} \tag{12}$$

$$= \frac{k}{1-k}\frac{\Pi_{0|NS} + \sum_{t=1}^{+\infty} SDF_{0,t}\Pi_t}{-\left(\Pi_{0|S} + \sum_{t=1}^{+\infty} SDF_{0,t}\Pi_t\right)} \tag{13}$$

With:

$$\Pi_t = p_t y_t - w_t n_t = p_t(1 - \frac{\epsilon - 1}{\epsilon})Y_t = \frac{p_t}{\epsilon}Y_t$$

$$\Pi_{0|\bullet} = p_0^s y_0^s - w_0 n_0^s$$

$$= p_0^s y_0^s - p_0 C_0^\sigma n_0^\psi n_0^s$$

$$= p_0^s y_0^s - p_0 Y_0^\sigma n_0^\psi \frac{y_0^s}{z_\bullet}$$

From now on, we will assume that we are in the case $a < 1$ at the equilibrium. This will be especially useful for taking the first derivative of a^*.

Given the renegotiation rule a^*, by the law of large numbers, the final good supply will be determined by the fraction ζ of firms that did not exit because they were allowed renegotiation:

$$Y_0 = \zeta(a^*)^{\frac{\epsilon}{\epsilon-1}} y_0^s \tag{14}$$

where $\zeta(a) \equiv \omega + (1-\omega)(k + (1-k)a)$. Finally, the labor demand is aggregated similarly as the measure of the remaining firms weighted by the inverse of their productivity:

$$n_0 = \gamma(a^*) y_0^s \tag{15}$$

where $\gamma(a) \equiv \omega + (1-\omega)(k + (1-k)a)\Delta_S$.

Dynamics

Note that, with investment and capital being absent from the model, and with pricing frictions being ineffective after the shock, monetary policy does not have a lasting effect, and real variables immediately go back to their steady-state levels. Monetary policy affects only time $t = 0$ real variables and only through i the interest rate between period 0 and period 1. Now, to fix things, we'll assume the central bank policy is composed of an endogenous part, and an exogenous non-persistent part, with the endogenous part defined as to peg the price level after the shock to its level pre-shock: $p_{t<0} = p_{t>0}$. Without loss of generality, we can normalize this target to 1. We'll then be interested in what happens to the equilibrium price if the central bank increases the exogenous part of the interest rate at time $t = 0$.

Simplified system

First, we'll proceed to simplify the above system of equations at time $t = 0$.

Denote Y_{SS} as the steady state quantity:

$$Y_{SS} = \left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{\sigma+\psi}}$$

Through household saving decisions, we have:

$$SDF_{0,t} = \beta^t \left(\frac{Y_t}{Y_0}\right)^{-\sigma} \frac{p_0}{p_t}$$

So, decomposing this into the Euler equations between subsequent periods, and using the fact that the economy is at its steady state from period 1:

$$SDF_{0,t} = \beta^t \left(\frac{Y_{SS}}{Y_0}\right)^{-\sigma} \frac{p_0}{p_t} = \frac{1}{1+i}\beta^{t-1}$$

Also, for $t \neq 0$:

$$\Pi_t = \frac{p_1}{\epsilon} Y_t = \frac{Y_{SS}}{\epsilon}$$

given the normalization on p. And for $t = 0$, the profits are:

$$\begin{aligned}
\Pi_{0|\bullet} &= p_0^s y_0^s - p_0 Y_0^\sigma n_0^\psi \frac{y_0^s}{z_\bullet} \\
&= Y_{SS} - p_0 Y_0^\sigma (\gamma(a^*) Y_{SS})^\psi \frac{Y_{SS}}{z_\bullet} \\
&= Y_{SS} - p_0 \zeta(a^*)^{\sigma \frac{\epsilon}{\epsilon-1}} \gamma(a^*)^\psi \frac{Y_{SS}^{\psi+\sigma+1}}{z_\bullet}
\end{aligned}$$

Where we used the fact that pre-contracted quantities y_0^s were agreed on in the "normal times" steady state, so $y_0^s = y_{SS}^s = Y_{SS}$.

So we can write the dynamic of the system at the time of the shock as:

$$\begin{aligned}
a^* &= \frac{k}{1-k} \frac{1 - p_0 \zeta(a^*)^{\sigma \frac{\epsilon}{\epsilon-1}} \gamma(a^*)^\psi Y_{SS}^{\psi+\sigma} + \frac{1}{1+i} \frac{1}{\epsilon(1-\beta)}}{-\left(1 - p_0 \Delta_S \zeta(a^*)^{\sigma \frac{\epsilon}{\epsilon-1}} \gamma(a^*)^\psi Y_{SS}^{\psi+\sigma} + \frac{1}{1+i} \frac{1}{\epsilon(1-\beta)}\right)} \\
p_0 &= \frac{1}{\beta(1+i)} \zeta(a^*)^{-\sigma \frac{\epsilon}{\epsilon-1}}
\end{aligned}$$

So finally, the equilibrium can be described by a scalar fixed point problem:

$$a^* = \frac{k}{1-k} \frac{1 - \frac{1}{\beta(1+i)} Y_{SS}^{\psi+\sigma} \gamma(a^*)^\psi + \frac{1}{1+i} \frac{1}{\epsilon(1-\beta)}}{-\left(1 - \frac{1}{\beta(1+i)} \Delta_S Y_{SS}^{\psi+\sigma} \gamma(a^*)^\psi + \frac{1}{1+i} \frac{1}{\epsilon(1-\beta)}\right)} \tag{16}$$

Existence, Uniqueness and differentiability of the equilibrium

For a standard value of ψ, $\psi = 1$, this fixed point problem just amounts to solving for the roots of a second-order polynomial in a^*. Far from thresholds where its discriminant changes signs, the roots for this polynomial are continuous and differentiable with respect to its coefficients. All that is then needed are restrictions for the polynomial to have a unique solution on the interval $[0, 1]$.

Now, we might as well experiment with $\psi = 0$. This will be an interesting case to consider as we will show later. Then a^* is absent from the RHS of (16), there is a unique equilibrium, a^* being directly pinned down by the parameters, including i. a^* is continuous and differentiable many times with respect to i, from the moment we don't hit the $a^* = 1$ case.

From now on we will assume a^* is unique and differentiable, at least piece-wise, so that we can take its first derivative hereafter.

3.4 Comparative statics

Assessing the micro-model conditions

The micro-model conditions will impose restrictions on the structural parameters, and hence on the comparative statics we can reasonably carry. Three conditions should be verified:

1. that at the equilibrium, $V_S = -T \leq 0 \leq V_{NS}$
2. Assumptions for Theorem 1:
   (a) Assumption 1: that $\Pi^C(a)$ is strictly increasing with a for $a \leq a^{Sep}$ and $a^{Sep} < a$.
   (b) Assumption 2: that $\Pi^C(a = 1) \leq \Pi^C(a = a^{Sep})$ taking p_0 as given.

Note that the assumptions are not formulated as expectations anymore as we consider population versions of the micro-model as we embed it in the general equilibrium. Let us write parametrically what 1., 2.(a) and 2.(b) lead to.

Condition 1.: First, consider the condition:

$$-T \leq 0 \leq V_{NS}$$

Let us derive more precisely what this condition entails:

$$-T \leq 0 \leq V_{NS} \Leftrightarrow -\frac{T}{Y_{SS}} \leq 0 \leq \frac{V_{NS}}{Y_{SS}}$$

$$\Leftrightarrow 1 - \frac{1}{\beta(1+i)} \Delta_S Y_{SS}^{\psi+\sigma} \gamma(a^*)^\psi + \frac{1}{1+i} \frac{1}{\epsilon(1-\beta)} \leq 0 \leq 1 - \frac{1}{\beta(1+i)} Y_{SS}^{\psi+\sigma} \gamma(a^*)^\psi + \frac{1}{1+i} \frac{1}{\epsilon(1-\beta)}$$

So condition 1. writes as:

$$0 \leq \Delta_S \frac{1}{\beta(1+i)} Y_{SS}^{\psi+\sigma} \gamma(a^*)^\psi - \frac{1}{1+i} \frac{1}{\epsilon(1-\beta)} - 1 \leq (\Delta_S - 1) \frac{1}{\beta(1+i)} Y_{SS}^{\psi+\sigma} \gamma(a^*)^\psi \tag{17}$$

Condition 2.(a):

Condition 2.(a) is equivalent to the extra profit from having one new firm staying afloat compared to the profit made from it exiting being higher than the cost from allowing renegotiation, so the transfer T.

More formally, given that:

$$\Pi^C(a < a^{Sep}) = p_0 \zeta(a)^{\frac{\epsilon}{\epsilon-1}} Y_{SS} - \zeta(a) Y_{SS} - (\zeta(a) - \omega - (1-\omega)k)T$$

$$\frac{\partial \cdot}{\partial a^*} = (1-\omega)(1-k) \left(p_0 \frac{\epsilon}{\epsilon - 1} Y_{SS}^{\frac{1}{\epsilon}} - Y_{SS} - T \right)$$

the condition 2.(a) for $a \leq a^{Sep}$ can be written:

$$p_0 \frac{\epsilon}{\epsilon-1} Y_{SS}^{\frac{\epsilon-1}{\epsilon}} Y_0^{\frac{1}{\epsilon}} - Y_{SS} > T \tag{18}$$

and a similar calculation for $a^{Sep} < a$ leads to:

$$p_0 \frac{\epsilon}{\epsilon-1} Y_{SS}^{\frac{\epsilon-1}{\epsilon}} Y_0^{\frac{1}{\epsilon}} - Y_{SS} > \frac{T}{1-\omega} \tag{19}$$

Condition 2.(b) Finally, the condition that the separating equilibrium is preferred to full renegotiation under uncertainty should be checked:

$$p_0 \zeta(a^{Sep})^{\epsilon/(\epsilon-1)} Y_{SS} - \zeta(a^{Sep}) Y_{SS} - (\zeta(a^{Sep}) - \omega)T \geq p_0(\omega(1-k) + (1-\omega))^{\epsilon/(\epsilon-1)} Y_{SS} - (\omega(1-k) + (1-\omega))(Y_{SS} + T)$$

And by rearranging terms, we obtain the following condition 2.(b):

$$Y_{SS} \left\{ p_0 \left(\zeta(a^{Sep})^{\epsilon/(\epsilon-1)} - (\omega(1-k) + (1-\omega))^{\epsilon/(\epsilon-1)} \right) - \left(\zeta(a^{Sep}) - (\omega(1-k) + (1-\omega)) \right) \right\} \geq T \left\{ \zeta(a^{Sep}) - 1 - \omega(1-k) \right\} \tag{20}$$

Sensitivity of the equilibrium to the interest rate

We'll take the first derivative with respect to i of both sides of the fixed point equation defining a^*:

$$\begin{aligned} aT &= \frac{k}{1-k}V_{NS} \\ \Leftrightarrow a\frac{\partial T}{\partial i} + T\frac{\partial a^*}{\partial i} &= \frac{k}{1-k}\frac{\partial V_{NS}}{\partial i} \end{aligned}$$

Now, we'll proceed by taking the first derivative of T and V_{NS} with respect to i:

$$\begin{aligned} \frac{\partial V_{NS}}{\partial i} &= Y_{SS}\left(\frac{1}{\beta(1+i)^2}Y_{SS}^{\psi+\sigma}\gamma(a^*)^{\psi} + -\frac{1}{\beta(1+i)}Y_{SS}^{\psi+\sigma}\psi\gamma(a^*)^{\psi-1}\frac{\partial\gamma(a^*)}{\partial a^*}\frac{\partial a^*}{\partial i} - \frac{1}{(1+i)^2}\frac{1}{\epsilon(1-\beta)}\right) \\ &= Y_{SS}\left(\frac{Y_{SS}^{\psi+\sigma}}{\beta(1+i)^2}\gamma(a^*)^{\psi} + -\frac{\psi\Delta_S(1-\omega)(1-k)Y_{SS}^{\psi+\sigma}}{\beta(1+i)}\gamma(a^*)^{\psi-1}\frac{\partial a^*}{\partial i} - \frac{1}{(1+i)^2}\frac{1}{\epsilon(1-\beta)}\right) \\ &= Y_{SS}\left(\left\{-\frac{\psi\Delta_S(1-\omega)(1-k)Y_{SS}^{\psi+\sigma}}{\beta(1+i)}\gamma(a^*)^{\psi-1}\right\}\frac{\partial a^*}{\partial i} - \left(\frac{1}{(1+i)^2}\frac{1}{\epsilon(1-\beta)} - \frac{Y_{SS}^{\psi+\sigma}}{\beta(1+i)^2}\gamma(a^*)^{\psi}\right)\right) \\ &\equiv Y_{SS}\left(-A\frac{\partial a^*}{\partial i} - (B-C)\right) \end{aligned}$$

Similarly:

$$\frac{\partial T}{\partial i} = -Y_{SS}\left(-\Delta_S A\frac{\partial a^*}{\partial i} - (B-\Delta_S C)\right)$$

Thus:

$$\frac{\partial a^*}{\partial i}\left((a+\frac{k}{1-k}\Delta_S)A + \frac{T}{Y_{SS}}\right) = -\left(\frac{k}{1-k}(B-C) + a(B-C\Delta_S)\right)$$

Its factor in the LHS of this expression being positive, $\frac{\partial a^*}{\partial i}$ is negative if and only if the RHS is negative:

$$\left(\frac{k}{1-k}(B-C) + a(B-C\Delta_S)\right) \geq 0$$

So simplifying this inequality, if and only if:

$$\frac{\beta}{1-\beta}(a+\frac{k}{1-k}) \geq (a\Delta_S+\frac{k}{1-k})(\epsilon-1)\gamma(a^*)^{\psi} \tag{21}$$

From now on, assume ψ is sufficiently close to 0 to discard the presence of the γ term. This amounts especially to discarding spillover effects from non-shocked to shocked firms through the labor market.

Now, at first glance, the inequality (**??**) is mainly parametric and potentially holds from reasonably tuning parameters, especially given $\beta \approx 1$. However, we have to verify that conditions for the well-definition of the micro-model are robust to the change of parameters needed. It turns out that condition 1. rules out the inequality to hold. However, as we will see below, we can afford to discard condition 1 without too much loss of generality, as a simple change in the micro-model makes condition 1 not rule out the inequality anymore. First, let us see how condition 1. in its present form rules out (**??**). Note that:

$$B - C = \frac{\frac{V_{NS}}{Y_{SS}} - 1}{1+i}$$

$$(C\Delta_S - B) = \frac{\frac{T}{Y_{SS}} + 1}{1+i}$$

Given $a = \frac{k}{1-k}V_{NS}/T$ checking the inequality amounts to asking:

$$\frac{V_{NS}}{Y_{SS}} - 1 \overset{?}{\geq} \frac{V_{NS}}{T}(\frac{T}{Y_{SS}} + 1)$$

But note that:

$$T(\frac{V_{NS}}{Y_{SS}} - 1) < V_{NS}(\frac{T}{Y_{SS}} + 1)$$

So the derivative with respect to a cannot be negative without violating the first condition. However, looking more deeply at why this is it the case, we found that a very light modification to the micro-model would lift this limitation. Indeed, consider the case where, for a given transfer T made to a firm, the receiving firm receives T if it was non-shocked, and $T - \delta$ with δ strictly positive (but never higher than T) if it was shocked. There is natural economic intuition for why a shocked firm would lose some of the transfer, whereas a non-shocked firm that operates normally will not. For instance, it could be that shocked firms lose financial advantages with banking intermediaries, having for example access to less competitive exchange rates. To keep the aggregates unchanged, assume that though it is lost by the firm, δ is recovered by the households. So given this situation, the transfer to make in the contract is not $-V_S$ anymore but is shifted up: $-V_S + \delta$. So this means a is $\frac{k}{1-k} \frac{V_{NS}}{-V_S+\delta}$, and this changes how a simplifies in the above calculation:

$$B - C = \frac{\frac{V_{NS}}{Y_{SS}} - 1}{1+i}$$

$$(C\Delta_S - B) = \frac{\frac{-V_S}{Y_{SS}} + 1}{1+i}$$

We need to ask:

$$\frac{V_{NS}}{Y_{SS}} - 1 \overset{?}{\geq} \frac{V_{NS}}{T} \left(\frac{T - \delta}{Y_{SS}} + 1 \right)$$

But note that now, after simplification, the LHS is $-T$, and the RHS is $V_{NS}(1 - \frac{\delta}{Y_{SS}})$, and the LHS is not necessarily lower than the RHS anymore.

With these arguments, we take as given for the rest of this section the fact that we can make the derivative of a with respect to i negative, by adjusting parameters while keeping valid the conditions for the micro-model. We move to the second part of the requirement for the existence of the reversal rate.

The reversal rate and the baseline interest rate level

Now, take the first derivative of the price p_0 with respect to i:

$$\frac{\partial p_0}{\partial i} = \frac{\partial}{\partial i}\left(\frac{1}{\beta(1+i)}\zeta(a^*)^{\frac{-\sigma\epsilon}{\epsilon-1}}\right)$$
$$= \frac{1}{\beta(1+i)^2}\zeta(a^*)^{\frac{-\sigma\epsilon}{\epsilon-1}-1}$$
$$\left(-\sigma\frac{\epsilon}{\epsilon-1}(1+i)(1-\omega)(1-k)\frac{\partial a^*}{\partial i} - \zeta(a^*)\right)$$

Thus, a necessary and sufficient condition for an extra tightening to fail in stabilizing inflation (i.e for this derivative to be positive) would be:

$$\frac{\partial a^*}{\partial i} \leq -\frac{(\omega + (1-\omega)(k + a^*(1-k)))}{\sigma\frac{\epsilon}{\epsilon-1}(1+i)(1-\omega)(1-k)} \tag{22}$$

So for the reversal rate to exist, we have to check a sign condition, (21), and an amplitude condition, (22). The sign condition checks that firm exit increases (a^* decreases) following a tightening of the rate. The amplitude condition verifies that this supply movement counterbalances the direct effect of i in decreasing p_0.

Now, for the rest of the analysis, we'll look at how the existence of the reversal phenomenon varies with the baseline interest rate i. We want to know whether reversal happens for higher baseline rates. First, note that: the minimal bound on the amplitude of $\frac{\partial a^*}{\partial i}$ in (22) is decreasing with i. Additionally, we can show that the amplitude of $\frac{\partial a^*}{\partial i}$ increases with i for i high enough. Indeed, under the approximation that $A \approx 0$ because of taking $\psi \approx 0$, we have that:

$$\left|\frac{\partial a^*}{\partial i}\right| \approx \frac{\tau\frac{1}{(1+i)^2}}{-1 + \frac{1}{1+i}(\frac{1}{\beta}\Delta_S Y_{SS}^{\psi+\sigma}\gamma(a^*)^{\psi} - \frac{1}{\epsilon(1-\beta)})}$$
$$\approx \frac{\tau}{-(1+i)^2 + \upsilon(1+i)}$$

where τ and υ are strictly positive functions of the parameters.

The amplitude is increasing with i if its denominator is decreasing with i. But the derivative of the denominator being $\upsilon - 2(1+i)$, we have that the denominator is decreasing (the amplitude is increasing) for i high enough ($i \geq \frac{\upsilon}{2} - 1$). So indeed: passed a threshold, the higher the i, the more likely the reversal.

4 General Equilibrium with a continuum of firms - TANK

4.1 New setting and Steady State

In this part, we will introduce household heterogeneity in the form of a stylized two-type economy, populated with a measure μ of pure investors and a measure $1-\mu$ of pure hand-to-mouth households. Our motivation for considering this economy is twofold. First, there is considerable evidence that a sizeable portion of wage-earners are either not savers, or show little sensitivity to interest rate movements (Kaplan et al. (2018)). Secondly, a key output from the previous part was that monetary tightening had as a second product the shifting up of labor supply, due to the shift in households' marginal utility of consumption in reaction to the tightening of rates. This corresponds to the term C in the above calculation of $\frac{\partial a^*}{\partial i}$). This channel led to a decrease in equilibrium wage that cushioned the shock, leading to $\frac{\partial a^*}{\partial i}$ to be potentially positive; thus there were questions on the sign of $B-C$ and on which effect of monetary policy would dominate.

We want to inquire now how this channel changes in the general equilibrium when we uncouple labor supply decisions from saving decisions.

The investors (from now on denoted with $._I$) maximize their intertemporal utility, with static utility given by:

$$U(C_I) = \frac{C_I^{1-\sigma}}{1-\sigma}$$

and discount the future with the parameter β.

The Hand-to-mouths (from now on denoted with $\cdot_H$) make only a static choice every period on how much to work:

$$U(C_H, n_H) = \frac{C_H^{1-\sigma}}{1-\sigma} - \frac{n_H^{1+\psi}}{1+\psi}$$

We use the same σ for both agents for the ease of use of the model.

So now, the household side is described by:

$$1 = \beta(1+i)\frac{p_t}{p_{t+1}}\left(\frac{C_{I,t+1}}{C_{I,t}}\right)^{-\sigma}$$
$$\frac{w_t}{p_t} = C_{H,t}^{\sigma} n_{H,t}$$

and the market clearing conditions now are:

$$Y_t = \mu C_{I,t} + (1-\mu) C_{H,t}$$
$$(1-\mu) n_{H,t} = \int n_t^s ds$$

The supply side stays identical.

As in the RANK model the steady state in the TANK model is characterized by constant quantities. From the supplier's pricing equation, we have at the steady state:

$$C_{H,SS}^{\sigma} n_{H,SS}^{\psi} = \frac{\epsilon - 1}{\epsilon}$$
$$\left(\frac{\epsilon - 1}{\epsilon} n_{H,SS}\right)^{\sigma} n_{H,SS}^{\psi} = \frac{\epsilon - 1}{\epsilon}$$
$$n_{H,SS} = \left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1-\sigma}{\sigma+\psi}}$$
$$c_{H,SS} = \left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1+\psi}{\sigma+\psi}}$$

And from the labor market clearing and the firms' symmetrical production functions:

$$Y_{SS} = n_{SS}$$
$$Y_{SS} = (1-\mu)\left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1-\sigma}{\sigma+\psi}}$$

From this we can derive the consumption of the investor:

$$
\begin{aligned}
c_{I,SS} &= \frac{1}{\mu}(Y_{SS} - (1-\mu)c_{H,SS}) \\
&= \frac{1-\mu}{\mu}(\frac{1}{\epsilon})\left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1-\sigma}{\sigma+\psi}}
\end{aligned}
$$

The parameters μ and ϵ govern the consumption that goes to the investor. ϵ has an ambiguous effect as higher elasticity of substitution means less market power and lower profit margins, but higher output.

Here, when the shock hits, similar to the previous framework, the economy will undergo a deviation from its steady state on impact, then come back to its steady state immediately the period after. Similarly to the previous framework, we will assume the endogenous part of the monetary policy rate is set such as to peg the economy after the shock at its pre-shock price, and we normalize this price. Now we will derive the deviations of the economy during the shock.

4.2 Reducing the model

Similarly to the previous case:

$$
\Pi_t = \frac{p_1}{\epsilon}Y_t = \frac{Y_{SS}}{\epsilon}
$$

given our normalization.

But current profits simplify differently than in the RANK:

$$
\begin{aligned}
\Pi_{0|\bullet} &= p_0^s Y_0^s - w_0 \frac{Y_0^s}{z_\bullet} \\
&= Y_{SS} - p_0 C_{H,0}^{\sigma} n_{H,0}^{\psi} \frac{Y_{SS}}{z_\bullet} \\
&= Y_{SS} - \frac{1}{\beta(1+i)} \frac{C_{H,0}^{\sigma}}{C_{I,0}^{\sigma}} \left(\frac{\gamma(a^*)}{1-\mu}\right)^{\psi} \frac{Y_{SS}^{\psi+1}}{z_\bullet} C_{I,SS}^{\sigma}
\end{aligned}
$$

On the other hand, we can derive how consumption will be shared between different types of households, from Hand-to-mouth's labour supply and the labor market clearing condition:

$$
\begin{aligned}
C_{H,0} &= \frac{w_0}{p_0} n_{H,0} \\
&= n_{H,0}^{\frac{\psi+\sigma}{1-\sigma}} n_{H,0} \\
&= n_{H,0}^{\frac{\psi+1}{1-\sigma}} \\
&= \left(\frac{n_0}{1-\mu} \right)^{\frac{\psi+1}{1-\sigma}} \\
&= \left(\frac{\gamma(a^*) Y_{SS}}{1-\mu} \right)^{\frac{\psi+1}{1-\sigma}}
\end{aligned}
$$

Thus:

$$
c_{I,0} = \frac{1}{\mu} \left(\zeta(a^*)^{\frac{\epsilon}{\epsilon-1}} Y_{SS} - (1-\mu) \left(\frac{\gamma(a^*) Y_{SS}}{1-\mu} \right)^{\frac{\psi+1}{1-\sigma}} \right)
$$

So finally, the equilibrium can be described by:

$$
a^* = \frac{k}{1-k} \frac{1 - \frac{1}{\beta(1+i)} \frac{C_{H,0}^{\sigma}}{C_{I,0}^{\sigma}} \left(\frac{\gamma(a^*)}{1-\mu} \right)^{\psi} \frac{Y_{SS}^{\psi}}{z_{\bullet}} C_{I,SS}^{\sigma} + \frac{1}{1+i} \frac{1}{\epsilon(1-\beta)}}{- \left(1 - \frac{1}{\beta(1+i)} \Delta_S \frac{C_{H,0}^{\sigma}}{C_{I,0}^{\sigma}} \left(\frac{\gamma(a^*)}{1-\mu} \right)^{\psi} \frac{Y_{SS}^{\psi}}{z_{\bullet}} C_{I,SS}^{\sigma} + \frac{1}{1+i} \frac{1}{\epsilon(1-\beta)} \right)} \tag{23}
$$

$$
\frac{c_{H,0}}{c_{I,0}} = \frac{\mu \left(\frac{\gamma(a^*) Y_{SS}}{1-\mu} \right)^{\frac{\psi+1}{1-\sigma}}}{\zeta(a^*)^{\frac{\epsilon}{\epsilon-1}} Y_{SS} - (1-\mu) \left(\frac{\gamma(a^*) Y_{SS}}{1-\mu} \right)^{\frac{\psi+1}{1-\sigma}}} \tag{24}
$$

4.3 Comparative statics in the TANK model

The first thing that should be noted is that now the marginal utility to consumption at time $t = 0$ does not simplify anymore in the expression for current profits, because of household heterogeneity, and the resulting decoupling of labor choice and intertemporal choice. In the RANK model, higher i was unambiguously shifting the wage down (taking ψ close to 0 to isolate the effect), because of i shifting down the price times the marginal utility of consumption of the representative agent. This same quantity was then pinning down the wage through the household's labor supply equation. In the TANK model, this now depends on how i moves the relative marginal utilities of consumption of the two types of agents, or equivalently, how it moves the relative consumption level of these agents. As equation 24 points to, how i is moving this ratio depends on how this ratio moves with a^*. But this is ambiguous, as it depends on the sign of:

$$\frac{\epsilon}{\epsilon-1}\frac{\gamma(a^*)}{\zeta(a^*)} - \frac{\psi+1}{1-\sigma}\Delta_S \tag{25}$$

(the ratio is increasing with a^* if and only if the latter quantity is negative).

Also, we note a change in how the derivative of the price with respect to i relates to the shift in output. As now the Euler equation relates to the marginal consumption of investors only, we have:

$$\begin{aligned}\frac{\partial p_0}{\partial i} &= \frac{\partial}{\partial i}\left(\frac{1}{\beta(1+i)}\frac{C_{I,SS}^{\sigma}}{C_{I,0}^{\sigma}}\right)\\ &= \frac{C_{I,SS}^{\sigma}}{\beta(1+i)^2}C_{I,0}^{-\sigma-1}\left(-\sigma(1+i)\frac{\partial C_{I,0}}{\partial i} - C_{I,0}\right)\end{aligned}$$

But we have:

$$C_{I,t} = \frac{\zeta(a^*)^{\frac{\epsilon}{\epsilon-1}}Y_{SS} - (1-\mu)\left(\frac{\gamma(a^*)Y_{SS}}{1-\mu}\right)^{\frac{\psi+1}{1-\sigma}}}{\mu}$$

And the sign of the derivative of this quantity with respect to a^* is also not straightforward to assess:

$$\frac{\epsilon}{\epsilon-1}\zeta(a^*)^{\frac{\epsilon}{\epsilon-1}-1}Y_{SS} - (1-\mu)\frac{\psi+1}{1-\sigma}\Delta_S\gamma(a^*)^{\frac{\psi+1}{1-\sigma}} \tag{26}$$

So from these two points, we see that heterogeneity here means that:

- the reversal rate, in the form we intuited in the RANK model, might not exist because the consumption of investors might increase in response to firm exit

- however, when the reversal rate exists, contrary to the previous case, the decoupling of labor decision and investment decision might contribute to reversal by amplifying the response of a^* to i, if the ratio of Hand-to-mouths' consumption to investors' consumption is increasing in a^*

Lastly, what seems striking overall for both derivatives 25 and 26 is the new role that σ plays. For σ close to 1, both these derivatives' signs get a definite sign: the ratio in (24) becomes increasing with a^*, and the consumption of investors becomes decreasing with a^*. Reversal is likely not to exist. This role played by σ was absent from the previous section and comes here from the hand-to-mouth nature of workers.

5 Conclusion

This paper provided a rich yet tractable framework to think about the constraints that monetary policy faces when hiking rates to stabilize prices during supply shocks. Through the lens of Force Majeure clauses, it micro-founded a channel from monetary policy to aggregate supply through firm exits. Our work revives the existing literature as it departs from the focus on intensive margin and from the reliance on credit constraints, which are completely absent from our model.

We showed how a reversal of monetary tightening could arise, and what were its determinants. The RANK model generates the intuitive fact that the higher the baseline interest rate, the more likely a further hike in interest rate to lead to reversal. The TANK model illustrates how the introduction of even basic heterogeneity leads to interesting reversal patterns, with at one extremity amplification of reversal arising and at another reversal being not possible.

A path for future work is to look at the robustness of our insights when adopting other assumptions for price determination. For instance, it is possible to write a model where the price level on impact is determined by a 0-profit condition for the final good firm. Another path to develop the reversal question is to build on macromodels with richer heterogeneity to achieve quantitatively sound results on the reversal.

The model for Force Majeure clauses we developed can be translated to answer other important questions. In follow-up work, we enquire what is the optimal fiscal policy to implement during large-scale Force Majeure events, such as pandemics. This work seeks to speak to the varying success of firm support policies conducted in the immediate aftermath of COVID-19's surge.

References

Abadi, J., Brunnermeier, M., and Koby, Y. (2023). The reversal interest rate. *American Economic Review*, 113(8):2084–2120.

Bernanke, B. and Gertler, M. (1989). Agency costs, net worth, and business fluctuations. *The American Economic Review*, 79(1):14–31.

Bernanke, B. S. and Blinder, A. S. (1992). The federal funds rate and the channels of monetary transmission. *The American Economic Review*, 82(4):901–921.

Bernanke, B. S. and Kuttner, K. N. (2005). What explains the stock market's reaction to federal reserve policy? *The Journal of Finance*, 60(3):1221–1257.

Bolton, P. and Whinston, M. D. (1993). Incomplete contracts, vertical integration, and supply assurance. *The Review of Economic Studies*, 60(1):121–148.

Dionne, G. and Doherty, N. A. (1994). Adverse selection, commitment, and renegotiation: Extension to and evidence from insurance markets. *Journal of Political Economy*, 102(2):209–235.

Elliott, M., Golub, B., and Leduc, M. V. (2022). Supply network formation and fragility. *American Economic Review*, 112(8):2701–47.

Grossman, G. M., Helpman, E., and Sabal, A. (2023). Resilience in Vertical Supply Chains. NBER Working Papers 31739, National Bureau of Economic Research, Inc.

Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary policy according to hank. *American Economic Review*, 108(3):697–743.

Khanna, G., Morales, N., and Pandalai-Nayar, N. (2022). Supply chain resilience: Evidence from indian firms. Working Paper 30689, National Bureau of Economic Research.

Kuttner, K. N. (2001). Monetary policy surprises and interest rates: Evidence from the Fed funds futures market. *Journal of Monetary Economics*, 47(3):523–544.

Laffont, J.-J. and Tirole, J. (1990). Adverse selection and renegotiation in procurement. *The Review of Economic Studies*, 57(4):597–625.

MacKay, A. (2022). Contract duration and the costs of market transactions. *American Economic Journal: Microeconomics*, 14(3):164–212.

Modigliani, F. and Miller, M. H. (1958). The cost of capital, corporation finance and the theory of investment. *The American Economic Review*, 48(3):261–297.

Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2):265–293.