Unemployment Insurance in Macroeconomic Stabilization with Imperfect Expectations[*]

Bence Bardóczy[†]   Joao Guerreiro[‡]

February 27, 2024

Preliminary – comments welcome!

Abstract

We study the power of unemployment insurance (UI) to stabilize short-run fluctuations in a state-of-the-art Heterogeneous-Agent New Keynesian (HANK) model featuring endogenous countercyclical income risk. Expectations are critical because higher UI generosity raises consumption, to a large extent, by lowering precautionary savings. If UI generosity is indexed to the unemployment rate, households must forecast the unemployment rate to anticipate the policy stance. At the microeconomic level, our model is rich enough to be consistent with evidence on income and consumption drops upon unemployment and the expiration of benefits. The model also features a general description of expectations combining incomplete information and long-memory diagnostic expectations. We estimate the model by matching evidence on the response of aggregates and survey measures of expectations to identified shocks. The estimated model implies that imperfect anticipation substantially affects the stimulative power of UI extensions: the expectations channel is responsible for over half of the response in the first year. We compare alternative ways of implementing UI policies. A UI extension that is announced directly is more stimulative in the very short run than one that is indexed to the unemployment rate.

Keywords: information frictions, bounded rationality, expectations surveys, unemployment insurance, heterogeneous agent New Keynesian models, stabilization policy.

[†]Federal Reserve Board. Email: bence.a.bardoczy@frb.gov.

[‡]University of California – Los Angeles. Email: jguerreiro@econ.ucla.edu.

1 Introduction

Unemployment insurance (UI) is an essential component of the social safety net. Temporary UI duration extensions are among the most commonly used fiscal-policy instruments to fight recessions. In the U.S., legislators have passed additional extensions on five separate occasions in the last 40 years. For example, during the Great Recession, the maximum duration of UI benefits increased from 26 weeks to 99 weeks. More recently, during the 2020 recession, unemployment benefits were again extended by 13 weeks. Despite the ubiquitous nature of UI extensions, their benefits and costs remain debated.

A recent literature emphasizes that a central channel by which UI operates is the households' *precautionary saving* motive (e.g., McKay and Reis 2016 and Kekre 2021). An increase in UI generosity boosts aggregate demand by reducing households' incentives to save in anticipation of unemployment spells. However, this modern literature assumes that people have full-information and rational expectations (FIRE). Assuming FIRE is consequential since precautionary saving depends on people's expectations regarding unemployment risk.

It is now well documented that survey data on beliefs show large deviations from FIRE (e.g., Coibion and Gorodnichenko 2012, 2015, Bordalo, Gennaioli, Ma and Shleifer 2020). For illustration, the left panel in Figure 1 shows the unemployment rate during the Great Recession alongside the consensus forecast for this variable at multiple horizons in the Survey of Professional Forecasters (SPF).[1] We highlight two main facts. First, SPF beliefs systematically under-forecasted the increase in the unemployment rate during the buildup phase. Second, following the peak of unemployment, forecasts lagged behind the decline in actual unemployment. To see why such mistakes may be relevant, note that Emergency Unemployment Compensation 2008 (EUC08) stipulated that UI benefits would be increased by an additional 13 weeks in case the unemployment rate increased above 6 percent. At the national level, this unemployment rate is reached in the third quarter of 2008. Interestingly, right until the quarter just before that, professional forecasters did not anticipate that the unemployment rate would ever cross the 6 percent threshold. This suggests that people may not have expected that Tier 3 would be activated.

When people must forecast the unemployment rate to infer UI generosity, expectations become critical to the success of UI extensions. In this paper, we are interested in understanding the power of UI extensions to stabilize business cycle fluctuations when people's expectations are taken directly from data, as opposed to forcing expectations to be FIRE.

---

[1]This figure does not represent definite proof for the failure of FIRE because new shocks could be realized at every point in time. The right panel shows that the same pattern of initial under-reaction followd by over-reaction is also present in the impulse responses of beliefs to an identified shock–the main business cycle shock of Angeletos, Collard and Dellas (2020).

Figure 1: Consensus Forecast of Unemployment Rate
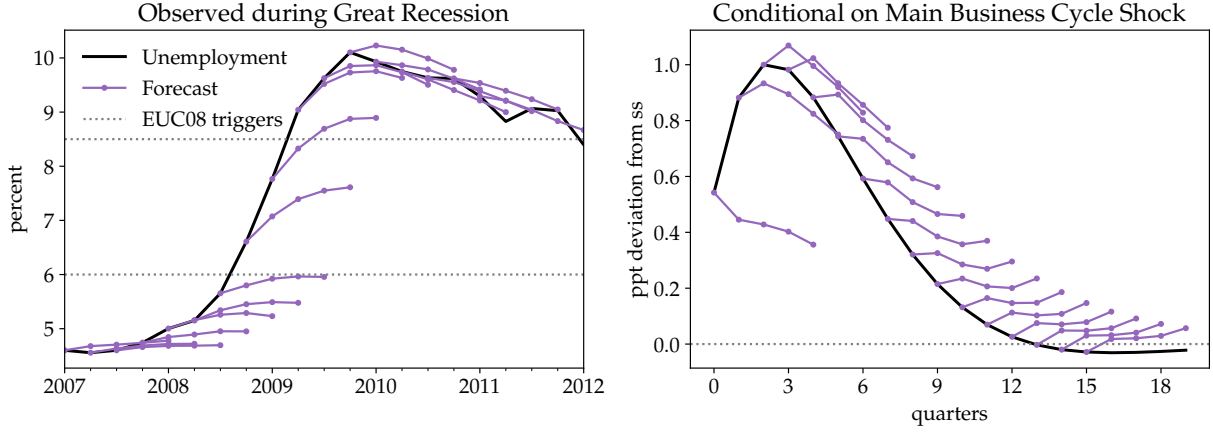
Notes. On both panels, the black line is based on the seasonally-adjusted civilian unemployment rate from the U.S. Bureau of Labor Statistics, and the purple line is based on the median unemployment forecast from the Survey of Professional Forcasters. On the left panel, we plot the level of the unemployment rate and forecasts between 2007Q1 and 2012Q1. On the right panel, we plot the impulse responses of these variables to the main business cycle shock (targeting unemployment) from Angeletos, Collard and Dellas (2020). We estimate the impulse responses using an ARMA-IV specification as in Angeletos, Huo and Sastry (2021). We scale the shock such that peak response of unemployment is 1 percentage point.

Illustrative model. We begin our analysis with an illustrative model that isolates the role of expectations in determining the equilibrium consumption response to UI benefit extensions. We work with a two-period setting which allows for an analytical solution. In both periods, workers can be either employed, earning labor income, or unemployed, earning UI benefits. An individual's unemployment shock is independent across periods. We consider a demand-induced recession in the second period, which increases households' precautionary saving motive in the first period. We compare two UI extension policies, that give rise to different expectations in period 0. The first is a policy rule that indexes unemployment benefits to the unemployment rate. The second is a policy of directly announcing unemployment benefits. We show that the difference between the responses of output in these two economies is given by the product of four terms:

$$dY_0^{\text{rule}} - dY_0^{\text{ann}} = -\mathcal{M} \cdot M_b \cdot \zeta_b \cdot \underbrace{(1-\lambda)dU_1}_{\text{forecast error}}, \tag{1}$$

where each term is: (1) the Keynesian-cross multiplier, $\mathcal{M} > 0$, (2) the partial-equilibrium response of aggregate demand to higher anticipated unemployment benefits, $M_b > 0$, (3) the elasticity of unemployment benefits to the unemployment rate, $\zeta_b > 0$, and finally (4) the forecast error in predicting the unemployment rate $(1-\lambda)dU_1$ where $1-\lambda$ denotes the *cognitive bias* and it is such that $dU_1^e = \lambda dU_1$, and $dU_1 > 0$ the increase in unemployment at time 1.

3

The relative performance of rules-based policy depends on whether beliefs under-react relative to FIRE ($\lambda < 1$) or over-react relative to FIRE ($\lambda > 1$). If individuals have FIRE, $\lambda = 1$, the output response is the same in both scenarios. Instead, if beliefs under-react compared to FIRE, individuals under-forecast the increase in UI benefits. It follows that the stabilization power of the policy is weaker. Instead, if beliefs over-react, the opposite happens. Individuals over-forecast future UI benefits, leading to a larger cut in precautionary savings and thus a milder recession. This model emphasizes that the anticipation of unemployment benefits is an important margin by which these policies transmit to consumption.

General framework. The simple model emphasizes the importance of getting expectations right in assessing the effects of UI extensions. To quantify the consequences of the empirical patterns of expectations, we provide a general framework that allows a more complete description of the economy and its actors and a more general description of their beliefs.

In section 3, we discuss a method to solve dynamic macroeconomic models under arbitrary beliefs about macroeconomic outcomes. This flexible method is based on the Sequence-Space Jacobian (SSJ) framework developed in Auclert, Bardóczy, Rognlie and Straub (2021) and extended to deviations from FIRE by Auclert, Rognlie and Straub (2020).[2] Building on their insights, we show that, to solve for aggregates, it is sufficient to describe how people respond to two additional objects: *forecast errors* and *forecast revisions*.

In the SSJ framework, FIRE is equivalent to perfect foresight. So, people make no forecast errors or revisions. It suffices to describe how people respond to the time-0 innovation, which forces the economy to deviate from a steady state. The Jacobians are sufficient statistics mapping changes in the path of endogenous and exogenous variables into the path of aggregate decisions of the agent block. For example, the consumption-real-interest-rate Jacobian $\mathcal{J}^{C,r}$ maps the change in real interest rates to the change in aggregate consumption of a household block. But, with general beliefs, people make mistakes in forecasting and may revise their expectations in the future. How individuals respond to these new objects can be computed directly from the FIRE Jacobian. The intuition for this result follows from the fact that because forecast errors and revisions are entirely unanticipated by the agents, then their response to these forecast updates is the same as their response to an unanticipated time-0 change. For example, the response of the household block to a forecast error in the time 1 real interest rate $r_1 - r_1^e$ is precisely the same as the agent would respond to a time 0 real interest rate shock r_0 under perfect foresight, $\mathcal{J}_{0,0}^{C,r}$.

Because it only uses the FIRE Jacobians, this method is very fast and easy to implement. We discuss how to implement a variety of popular models of deviations from FIRE.[3] More importantly, the framework allows us to work with arbitrary expectations. We leverage this fact by working directly with empirically measured expectations. This allows us to quantify the impact

---

[2] Auclert, Rognlie and Straub (2020) show how to manipulate the FIRE Jacobian to implement sticky expectations (Mankiw and Reis 2002, Carroll, Crawley, Slacalek, Tokuoka and White 2018), cognitive discounting (Gabaix 2020), and dispersed information (Angeletos and Huo 2021). They also discuss how this idea can be extended to other models.

[3] See Appendix B.3.

of imperfect expectations on the power of UI extensions in stimulating aggregate demand without imposing any extra assumptions on the belief formation model. Effectively, this method allows us to sidestep the issue of choosing among the "wilderness" of alternative models of belief formation, a common criticism of this literature going back to Sims (1980) and Sargent (2001).

Quantitative framework and results. Equipped with a framework to solve and analyze dynamic models with arbitrary deviations from FIRE, we refine the analytical results in (1). We need four objects. First, the dynamics of forecast errors and revisions about the unemployment rate. Second, a UI extension policy that indexes duration to the unemployment rate. Third, a model of households that maps beliefs about UI duration into aggregate demand. Fourth, a model of the macroeconomy that maps changes in aggregate demand into equilibrium outcomes.

To obtain empirically relevant forecast errors, we estimate the impulse responses of the unemployment rate and its forecasts at different horizons to an identified aggregate shock. We measure expectations as the consensus forecast from the SPF. Our identified shock is the main business cycle (MBC) shock of Angeletos, Collard and Dellas (2020). The MBC shock is a natural choice because it accounts for the largest share of unemployment fluctuations over the business cycle by construction.

We implement automatic UI extensions via a policy rule that indexes the UI expiration probability to the equilibrium unemployment rate. We calibrate the semi-elasticity in the rule, ζ_b, to match the ratio of UI extensions in the EUC08 policy to the rise in the unemployment rate during the Great Recession. The calibrated rule implies that a one percentage point increase in the unemployment rate triggers about a one-quarter increase in average UI duration.

We embed this policy rule in a New Keynesian model with incomplete markets, heterogeneous households, and search and matching frictions. Our model incorporates many features that have been emphasized in modern models of social insurance (McKay and Reis, 2016, Kekre, 2021). Notably, it features intertemporal optimization by risk-averse, borrowing-constrained households; heterogeneity in marginal propensities to consume (MPC); endogenous unemployment risk; and nominal rigidities. We estimate our model following the procedure popularized by Christiano, Eichenbaum and Evans (2005) and recently extended to an heterogeneous-agents environment by Auclert, Rognlie and Straub (2020). First, we calibrate the model's steady state to match a list of relevant moments, including MPCs. Second, we estimate the remaining structural parameters by matching the empirical impulse responses of select aggregate variables. Importantly, this estimation exercise can be performed using directly the expectations observed in the data in response to the identified shocks.

Our estimated model implies that perceived UI duration is more important for aggregate stabilization than actual UI duration. UI extensions raise income only for those workers who experience a job loss and stay eligible thanks to the extension. Most households remain employed even in deep recessions; and so, for them, only perceived UI duration matters, which affects their precautionary saving. So expectations are crucial in assessing the effectiveness of the policy. We

show that the policy is less effective in the short run relative to FIRE benchmark. This finding is a direct consequence of the initial under-reaction observed in Figure 1. However, after the peak of the recession, expectations turn overly pessimistic. This pattern of delayed over-reaction implies that the policy becomes even more effective under the estimated beliefs than under FIRE.

We use our model to quantify the impact of UI extensions on equilibrium unemployment and consumption relative to a counterfactual scenario in which UI duration was constant. In order to run counterfactuals, we describe and estimate a model of belief formation which combines noisy information with long-memory diagnostic expectations. We show that, at the onset of the recession, the policy reduces the unemployment rate by 0.4 percentage points and increases aggregate consumption by 0.6 percentage points, while in FIRE the same policy would have reduced the unemployment rate by 0.7 percentage points and increased consumption by 1 percentage point. It follows that the initial belief under-reaction makes this type of policy almost half as effective as would be predicted by models with full-information and rational expectations. However, due to the pattern of delayed over-reaction, the impact of the policy on aggregates is hump-shaped in our model (instead, with FIRE, the peak effectiveness happens immediately). In our model, the peak effectiveness of the policy leads to a reduction of 0.5 percentage points in unemployment and an increase in consumption of almost 1 percentage point.

Finally, we use our model to assess the relative efficacy of different forms of policy communication. In particular, as in Bianchi-Vimercati, Eichenbaum and Guerreiro (2021), we evaluate the stabilization power of announcing the UI duration directly to people rather than implementing as a contingent rule. We conclude that announcing the policy directly can be very stimulative in the very short run, but may lack efficacy later in the recession as expectations turn overly pessimistic.

Relationship to the literature. Our paper contributes to an extensive literature analyzing the consequences of macroeconomic shocks and policies without FIRE and exploiting survey data to calibrate the expectational components of macro models. We share the interest in analyzing these questions in the context of Heterogeneous-agent New Keynesian models (HANK) with the recent contributions by Farhi and Werning (2019), Farhi, Petri and Werning (2020), Auclert, Rognlie and Straub (2020), Pappa, Ravn and Sterk (2023), Dobrew, Gerke, Giesen and Röttger (2023), and Guerreiro (2022). These papers consider parametric models of bounded rationality. We deviate from their contributions in two ways. First, we study the effects of UI extensions on the economy. Second, we discuss a method that allows us to quantify the impact of deviations from FIRE in a non-parametric way, directly exploiting the data coming from surveys of expectations. This allows us to sidestep the discussion of choosing a particular model of deviation from FIRE.

The idea that unemployment expectations are important for business cycles goes back to Carroll (1992). Our model includes a list of features (incomplete markets, nominal rigidities, and suboptimal monetary policy) which have been found important in this line of research since then. Christiano, Eichenbaum and Trabandt (2016) show that nominal rigidities and constraints on monetary policy adjustment tend to reverse the contractionary effects of UI extensions in Krusell,

6

Mukoyama and Şahin (2010), Nakajima (2012), and Mitman and Rabinovich (2015, 2019). Furthermore, Kekre (2021) emphasizes how these mechanisms can be complemented by the stimulus effect arising from the direct redistribution across workers with different marginal propensities to consume and the impact of reducing precautionary savings motives. Relatedly, Bilbiie, Primiceri and Tambalotti (2022) find that cyclical income risk and precautionary saving behavior substantially amplify business cycles. However, this literature has worked exclusively with FIRE. Our paper contributes a new perspective on the quantitative relevance of the different mechanisms when beliefs accord to the survey evidence.

In a closely related paper, Fernandes and Rigato (2022) study UI in a model where households have present-biased preferences. Present bias reduces the responsiveness of precautionary saving to UI extensions. However, they maintain the assumption of full-information rational expectations, making their contribution complementary to ours.

Outline. The structure of the paper is as follows. Section 2, analytical model. Section 3, general framework with propositions. Section 4, quantitative model and results. Section 5 concludes.

2 Illustrative model

We start with an analytical demonstration that imperfect expectations affect the power of unemployment insurance (UI) extensions to stabilize business cycles. We consider a simple two-period environment. We engineer a recession in period 1, which triggers precautionary responses in period 0. Then, we analyze how equilibrium output at time 0 depends on households' expectations and the implementation of UI. Appendix A contains detailed derivations and proofs.

2.1 Setup

Consider a two-period model, $t = 0, 1$. The economy is populated by a measure one of households, a representative firm, and a government. The sequence of events within the two periods is the same. First, the representative firm randomly hires a fraction of households. Second, production takes place and households make a consumption-saving decision.

Firm. A competitive firm produces a final good Y_t from labor N_t according to the production function

$$Y_t = N_t \tag{2}$$

The only cost of production is the real wage bill $w_t N_t$ paid to workers. In equilibrium, $w_t = 1$ and the firm hires just enough workers to meet aggregate demand while making zero profit.

Policy. The government runs a balanced budget

$$\tau_t = (1 - N_t)b_t \tag{3}$$

7

We assume that the government specifies a rule for automatic adjustment of UI benefits to the contemporaneous unemployment rate $b_t = b - \zeta_b N_t$.

We specify the different implementations of unemployment benefits b_t below in the context of the business cycle stabilization experiment.

We assume that monetary policy target and implements a level for nominal GDP:

$$P_t C_t = M_t \qquad (4)$$

where P_t is the price level, C_t is aggregate consumption, and M_t is the nominal GDP target. We assume that prices are fully rigid and normalize the price level to one, $P_t \equiv 1$. The monetary authority sets the level of nominal GDP at time 1, M_1, and the real rate between periods 0 and 1, r. Let M_0 adjust to support the equilibrium given exogenous monetary policy (r, M_1).

In this simple model, we consider an exogenous shock to time-1 nominal GDP M_1. The combination of sticky prices and the nominal GDP equation (4) implies that these shocks also affect aggregate quantities. As a result, these assumptions allow us to consider demand shocks in this simple two-period model.

Households. In period t, $N_t \in [0, 1]$ of households are employed. The remaining $1 - N_t$ households are unemployed. The probability that an individual household is employed is the same for all workers and equal to the employment rate N_t. Employed workers earn real wage $w_t = 1$. Unemployed workers receive real benefits $b_t \in (0, 1)$, financed by a lump-sum tax τ_t levied on all households. Once their employment status for the current period, $e_t \in \{0, 1\}$, is determined, households choose consumption c_t and savings a_t in a non-contingent bond with real return r to maximize their anticipated life-time utility

$$u(c_0) + \beta u(c_1) \qquad (5)$$

subject to period budget constraints

$$c_t + a_t = (1 + r)a_{t-1} + e_t w_t + (1 - e_t)b_t - \tau_t \qquad (6)$$

and borrowing constraints $a_t \geq 0$ for $t = 0, 1$. We assume that $u(\cdot)$ is smooth, increasing, concave, and has a positive third derivative, i.e. households are prudent in the sense of Kimball (1990).

At time 0, households may not have perfect foresight of the endogenous variables N_1, b_1, τ_1, and hence even of their own consumption c_1. Let $E[N_1], E[b_1]$ and $E[\tau_1]$, denote their expectations for aggregate labor demand N_1, government benefits b_1, and the tax τ_1, respectively. For simplicity, we assume that the household expectations of taxes and benefits are consistent with knowledge of the government's rules for benefits and the budget constraint.[4] We also assume that all households have the same beliefs and do not consider uncertainty.[5]

---

[4]We relax this assumption in the quantitative model.

[5]Since we focus on the first-order response of this economy around a non-stochastic equilibrium, disregarding un-

Prudence and market incompleteness implies that households have precautionary saving motive in period 0 against unemployment risk in period 1. We can see this from the Euler equation

$$u'(c_0) \geq \beta(1+r) \left[E[N_1] \cdot u'\big(\underbrace{1 - E[\tau_1] + (1+r)a_0}_{c_1 \text{ if employed}} \big) + (1 - E[N_1]) \cdot u'\big(\underbrace{E[b_1] - E[\tau_1] + (1+r)a_0}_{c_1 \text{ if unemployed}} \big) \right] \tag{7}$$

As is standard in models at the zero liquidity limit (e.g., Werning 2015), we assume that at least one Euler equation holds with equality. As we show in appendix A.1, this will be the employed workers' Euler equation, because they have a stronger incentive to save in period 0. Then, (7) implies that the consumption of employed workers in period 0, $c_0(E)$, is increasing in the expectations for employment in period 1, $E[N_1]$.

Equilibrium. To assess the impact of expectations on the equilibrium, we make no further assumptions on how expectations are generated. So, we define a temporary equilibrium.

Given initial assets a_{-1}, exogenous variables $\{b_t, r, M_1\}$, and beliefs $\{N_1^e, b_1^e, \tau_1^e\}$, a temporary equilibrium is a collection of prices $\{w_t\}$ and allocations $\{c_t^E, c_t^U, N_t, \tau_t, M_0\}$ such that the representative firms optimizes, households optimize, government budget is balanced, the cash in advance constraint is satisfied, goods market clears

$$Y_t = C_t = N_t c_t^E + (1 - N_t)c_t^U \tag{8}$$

and asset market clears

$$0 = A_t = N_t a_t^E + (1 - N_t)a_t^U \tag{9}$$

The formal derivation of the model solution is relegated to appendix A.1. In the zero liquidity limit, the model is purely forward-looking. So the time-1 equilibrium is independent of time-0 outcomes, including the expectations that households hold in period 0. However, since employed workers are on the Euler equation, their expectations are relevant for equilibrium in period 0. As such, the model isolates the effect of imperfect anticipation of benefits in general equilibrium (GE).

2.2 Macroeconomic stabilization

We demonstrate that deviations from FIRE affect the power of unemployment benefit extensions to stabilize aggregate demand. To this end, we induce a recession at time $t = 1$ and characterize the first-order change in equilibrium at time $t = 0$ from anticipating the recession.

The recession originates in a decrease in time-1 nominal GDP, $dM_1 < 0$, that translates one-to-one into lower employment, $dN_1 = dM_1$. In response, employed households will try to save more in period 0 according to the Euler equation (7). Since they cannot save in equilibrium, their time-0 consumption has to fall to dissuade them from saving. Thus a recession arises endogenously in

---

certainty does not impact our results.

period 0. The recession's severity depends on the strength of households' precautionary saving motive which depends on expected unemployment benefits.

The news of the shock changes household expectations of employment at time 1, $E[dN_1]$. Under full information rational expectations (FIRE), expectations would be correct, i.e., $E[dN_1] = dN_1$. More generally, people may make forecast errors $E[dN_1] - dN_1$. The change in the unemployment rate at time 1 leads household expectations of unemployment benefits to change: $E[db_1] = -\zeta_b E[dN_1]$.

We measure the efficacy of the automatic stabilizer by the reduction in the response of output relative to a counterfactual economy without the policy. We also normalize the by the change in benefits db_1. So, efficacy δ is defined as follows:

$$\delta \equiv \frac{dY_0^{db_1=0} - dY_0}{db_1}$$

Our main result, in 1, relates the efficacy of the policy for general beliefs to that that would be obtained under FIRE.

Proposition 1. *Let δ and δ^{FIRE}, denote the efficacy of UI extensions in our benchmark model and under full-information and rational expectations, respectively. Then,*

$$\delta = \delta^{FIRE} - \frac{1}{1 - \mathrm{MPC}_0} M_1^b (1 - \lambda), \tag{10}$$

where MPC_0 denotes the time-0 marginal propensity to consume, M_1^b denotes the propensity to consume out of announced benefits, and $\lambda \equiv E[db_1]/db_1$ denotes people's cognitive-bias.

Equation (10) shows how deviations from FIRE affect the efficacy of UI. Under FIRE ($\lambda = 1$), households forecast the unemployment rate and benefits perfectly. If $\lambda \neq 1$, households erroneously forecast the increase in the unemployment rate at time 1. So, they make mistakes in forecasting the increase in generosity of unemployment benefits. In other words, their misperception of tomorrow's unemployment rate also translates into a misperception of the future policy stance. Their forecast error is given by their cognitive bias $1 - \lambda$. Today's impact of this cognitive bias is mediated by two forces: the Keynesian cross term $1/(1 - \mathrm{MPC}_0) > 0$ and the partial-equilibrium effect of time-1 redistribution on aggregate demand today $M_1^b > 0$. Naturally, the higher any of these terms, the higher is the impact of deviations from FIRE on the efficacy of UI stabilization.

But, is the stabilization power of FIRE higher or lower than under full-information and rational expectations? The answer to this question crucially depends on whether λ is larger or smaller than 1. In other words, it depends on whether expectations react more or less than under FIRE. If household beliefs under-react relative to FIRE, $\lambda < 1$, forecast mistakes make the rules-based policy less effective than the instrument-announcement policy. Instead, if household beliefs over-react relative to FIRE, $\lambda > 1$, forecast mistakes make the rules-based policy more effective than the instrument-announcement policy.

10

Do beliefs under-react or over-react to innovations in fundamentals? This question has been the focus of an extensive empirical literature looking at survey evidence, but a consensus has not been reached. For instance, Coibion and Gorodnichenko (2012, 2015) find evidence of belief under-reaction. This finding is consistent with models of rational inattention or information rigidities, as in Sims (2003), Woodford (2001), Carroll (2003), Mankiw and Reis (2002), or Gabaix (2020). Instead, Bordalo, Gennaioli, Ma and Shleifer (2020) find evidence of belief over-reaction, which is consistent with models of diagnostic expectations and overextrapolation as in Bordalo, Gennaioli and Shleifer (2018). More recently, Angeletos, Huo and Sastry (2021) find evidence of initial under-reaction and a pattern of delayed over-reaction. Given the central importance of expectations in our analysis, in Section 3, we present a framework that can accommodate arbitrary deviations from FIRE. In Section 4, we discuss how this general framework can be used to accomodate a general description of expectations. We develop a model of expectations which can reconcile the empirical findings based on surveys of expectations and exploit these data to estimate our economy.

3 A framework for dynamic models with imperfect expectations

Next we lay out a framework of dynamic decision making with imperfect expectations. Our framework has two components. First, a model of how actions evolve given any expectations. Second, a model of how expectations are formed from observations. In appendix B, we map many popular models of bounded rationality and information frictions into our framework.

3.1 Dynamic decisions with general deviations from FIRE

Consider a forward-looking agent who chooses an output Y_t over periods $t = 0, 1 \ldots, T - 1$. Let the vector $\mathbf{Y} \in \mathbb{R}^T$ denote the path of the output. For ease of exposition, let every object (parameters, initial-, and terminal conditions) that matters for the decision be fixed and known to the agent except the path of a single univariate input $\mathbf{X} \in \mathbb{R}^T$. The extension to multiple time-varying inputs is straightforward.

Auclert, Bardóczy, Rognlie and Straub (2021) cast such dynamic decision problems as a mapping between sequences

$$\mathbf{Y} = f(\mathbf{X}) \tag{11}$$

Their SSJ method computes the Jacobian $\mathcal{J} \in \mathbb{R}^{T \times T}$ then computes impulse responses to any shock $d\mathbf{X}$ via matrix multiplication, $d\mathbf{Y} = \mathcal{J} d\mathbf{X}$.[6] The representation (11) is valid under two assumptions. First, certainty equivalence with respect to $\mathbf{X}$. When the agent chooses Y_t, she considers only her time-t expectations $\mathbf{X}^{e,t} \in \mathbb{R}^T$, not the entire distribution of $\mathbf{X}$. Second, perfect foresight (FIRE) with respect to $\mathbf{X}$. The agent's expectations are correct, $\mathbf{X}^{e,t} = \mathbf{X}$.

---

[6]The Jacobian is computed at a baseline path $\bar{\mathbf{X}}$, typically a constant path corresponding to the steady state $\bar{\mathbf{Y}} = f(\bar{\mathbf{X}})$. So the shock $d\mathbf{X} = \mathbf{X} - \bar{\mathbf{X}}$ and the impulse response $d\mathbf{Y} = \mathbf{Y} - \bar{\mathbf{Y}}$ are both deviations from the baseline path.

We are interested in a generalization of this setup which relaxes the assumption of FIRE. We retain certainty equivalence, so only the mean expectation matters. However, expectations may not be correct and may evolve over time. In period 0, the agent expects a path $\mathbf{X}^{e,0}$; in period 1, she expects a path $\mathbf{X}^{e,1}$; and so on. Each vector $\mathbf{X}^{e,\tau} = \begin{bmatrix} X_0^{e,\tau} & X_1^{e,\tau} & ... & X_T^{e,\tau} \end{bmatrix}'$ captures the beliefs that the agent holds at time τ about the variable X at every date. We assume that the agent observes current and past realizations (or, alternatively, all current realizations and the state variables for their individual decision making), and also assume that the agent does not foresee their future forecast errors (i.e., they are naive). So $X_t^{e,\tau} = X_t$ for all $t \leq \tau$. This ensures that the agent does not violate any constraints. In sum, relaxing FIRE implies that we have to keep track of the entire history of expectations, $\mathbf{X}^{e,t}$ for all t. Formally,

$$\mathbf{Y} = g\left(\mathbf{X}, \{\mathbf{X}^{e,t}\}_t\right) \tag{12}$$

Conceptually it is clear that if we could compute all the Jacobians of $g(\bullet)$, we could compute linearized impulse responses. But the domain of $g(\bullet)$ is $\mathbb{R}^{T+T\times T}$, a much larger space than the domain of $f(\bullet)$ which is just $\mathbb{R}^T$. Is this approach viable in practice? Propositions 2 and 3 show that it is. The key idea is to manipulate the FIRE Jacobian $\mathcal{J}$ to capture the responses to forecast errors. This insight appears in Auclert, Rognlie and Straub (2020), who implemented specific deviations from FIRE via Jacobian manipulation.[7] Propositions 2 and 3 do the same for general deviations from FIRE, using the familiar concepts of forecast errors and forecast revisions.

Proposition 2 handles the special case of non-rational but time-invariant expectations $d\mathbf{X}^e \neq d\mathbf{X}$. An example of this is level-k thinking. The total response $d\mathbf{Y}$ is the sum of two effects. First, the response to the expected part of the shock. Second, the responses to the forecast errors that the agent observes along the way. The key new object is the forecast-error Jacobian, $\mathcal{E}$, that captures the second effect. Column s of $\mathcal{E}$ can interpreted as the impulse response to the forecast error in dX_s which the agent learns in period s. Constructing $\mathcal{E}$ is straightforward. It is a lower diagonal matrix whose columns are shifted versions of the first column of $\mathcal{J}$. The intuition is that observing a forecast error in period t is equivalent to observing an unexpected shock in period 0. The formal proof is in appendix B.1.

Proposition 2. *Assuming constant beliefs $X_{t+h}^{e,t} = X_{t+h}^{e,0}$ for all $t, h > 0$, the linearized impulse response $d\mathbf{Y}$ to an arbitrary shock $d\mathbf{X}$ is given by*

$$d\mathbf{Y} = \mathcal{J} \underbrace{d\mathbf{X}^{e,0}}_{forecast} + \mathcal{E} \underbrace{\left(d\mathbf{X} - d\mathbf{X}^{e,0}\right)}_{forecast\ error} \tag{13}$$

---

[7]Appendix D.3 of Auclert, Rognlie and Straub (2020) provides recipes to implement sticky expectations, cognitive discounting, and dispersed information.

*where the **forecast-error Jacobian** $\mathcal{E}$ is given by*

$$\mathcal{E} = \begin{bmatrix} \mathcal{J}_{0,0} & 0 & \cdots & 0 \\ \mathcal{J}_{1,0} & \mathcal{J}_{0,0} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{J}_{t,0} & \mathcal{J}_{t-1,0} & \cdots & \mathcal{J}_{0,0} \end{bmatrix} \tag{14}$$

Proposition 3 handles the general case of time-variant expectations. The new element is that observing the forecast error $dX_t - dX_t^{e,t-1}$ may cause the agent to update her expectations for all future periods. Capturing this effect is most straightforward if we work with forecast revisions $d\boldsymbol{X}^{e,h} - d\boldsymbol{X}^{e,h-1}$ instead of forecast errors $d\boldsymbol{X} - d\boldsymbol{X}^{e,h}$. The Jacobians that act on forecast revision vectors are simply shifted versions of the FIRE Jacobian $\mathcal{J}$. The intuition is that a forecast revision for periods $t, \dots, T-1$ is equivalent to observing an unanticipated shock for periods $0, \dots, T-t$. The formal proof is in appendix B.2.

Proposition 3. *Assuming time-variant beliefs $\boldsymbol{X}^{e,t}$, the linearized impulse response $d\boldsymbol{Y}$ to an arbitrary shock $d\boldsymbol{X}$ is given by*

$$d\boldsymbol{Y} = \mathcal{J} \underbrace{d\boldsymbol{X}^{e,0}}_{\text{initial forecast}} + \sum_{h \geq 1} \mathcal{R}_h \underbrace{\left(d\boldsymbol{X}^{e,h} - d\boldsymbol{X}^{e,h-1} \right)}_{\text{forecast revision}} \tag{15}$$

*where the **forecast-revision Jacobian** $\mathcal{R}_h$ for any $h > 1$ is given by*

$$\mathcal{R}_h = \begin{bmatrix} 0 & \mathbf{0}'_h \\ \mathbf{0}_h & \mathcal{J} \end{bmatrix}$$

Application to heterogeneous-agent models. Propositions 2 and 3 apply to heterogeneous-agent models in which $Y_t = \int y_t dD_t$ is an aggregate of individual decisions y_t for some non-trivial, time-varying distribution D_t. However, we need to impose restrictions on belief heterogeneity. In the exposition above, we assume that everyone has the same beliefs. More generally, this framework can be directly used even if expectations are heterogeneous as long as they are uncorrelated with other idiosyncratic characteristics in the cross-section. For this purpose, we redefine $X^{e,t}$ as the cross-sectional average expectation.

It is also possible to use this framework to allow for meaningful belief disagreement as long as beliefs are with permanent individual characteristics. In this case, one has to set up a heterogeneous-agent block for each permanent type, and apply the propositions type by type. Guerreiro (2022) follows this approach in his study of disagreements over the business cycle.

3.2 A flexible model of expectations

Propositions 2 and 3 enable us to compute linearized impulse responses to any shock $d\mathbf{X}$ given the path of expectations $\{d\boldsymbol{X}^{e,t}\}_t$ conditional on the same shock.[8] In some cases, the response

---

[8]Recall that deviations $d\mathbf{X}$ and $\{d\boldsymbol{X}^{e,t}\}_t$ are all relative to the paths around which one wishes to compute the Jacobian.

of expectations may be estimated directly. We'll do so in section 4.6 with respect to unemployment. Another route is to impose a model of expectation formation. In this subsection, we present a tractable yet flexible specification that nests many popular models including: (1) sticky expectations (Mankiw and Reis, 2002, Carroll, Crawley, Slacalek, Tokuoka and White, 2018), (2) noisy-information and rational expectations (Angeletos and Huo, 2021), (3) cognitive discounting (Gabaix, 2020), (4), sparsity (Gabaix, 2014, 2016, Guerreiro, 2022), (5) shallow reasoning (Angeletos and Sastry, 2021), (6) finite planning horizons (Woodford, 2018), (7) adaptive expectations (Cagan, 1956, Friedman, 1957), (8) diagnostic expectations (Bordalo, Gennaioli and Shleifer, 2018, Bianchi, Ilut and Saijo, 2021), (9) noisy-information diagnostic expectations (Bordalo, Gennaioli, Ma and Shleifer, 2020), among others. Appendix B.3 discusses how to map each of these models into our framework.

Propositions 2 and 3 deal with linear mappings in sequence space. So it's natural for us to model expectations in the same way. The most general linear sequence-space model of expectations we can write down—given a single time-variant input $d\mathbf{X}$—is a sequence of matrices $\Lambda_t \in \mathbb{R}^{T \times T}$ that map realized outcomes $d\mathbf{X} \in \mathbb{R}^T$ into time-t expectations $d\mathbf{X}^{e,t} \in \mathbb{R}^T$ according to

$$dX^{e,t} = \Lambda_t dX \tag{16}$$

We maintain the assumption that expectations of current and past realizations of $d\mathbf{X}$ are correct. This implies that the upper-left block of Λ_t is the identity matrix

$$\Lambda_t = \begin{bmatrix} \mathbf{I}_{t \times t} & \cdots \\ \vdots & \ddots \end{bmatrix} \tag{17}$$

Equation (16) looks simple but it can capture rich theories of expectation formation. It can account for an understanding of the data generating process as well as for updating of priors in light of new observations. In our expository environment, $d\mathbf{X}$ is the only input the agent has to form expectations about. So it's not restrictive to assume that $d\mathbf{X}^{e,t}$ depends only on the realized path of $d\mathbf{X}$ itself. In richer environments with multiple inputs, one may introduce additional linear terms, allowing the agent to think about cross-equation restrictions directly. For example, expectations of unemployment benefits and income taxes may be related via an understanding of the government budget constraint.

Corollaries 1 and 2 substitute (16) into propositions 2 and 3. The bracketed terms can be interpreted as the Jacobians of the non-FIRE decision problem represented by the function $g\left(\mathbf{X}, \{\mathbf{X}^{e,t}\}_t\right)$. These non-FIRE Jacobians account for the direct effect of $d\mathbf{X}$ on $d\mathbf{Y}$ as well its indirect effect through $\{\mathbf{X}^{e,t}\}_t$. Crucially, they're still $T \times T$ matrices just like the FIRE Jacobians of $f(\mathbf{X})$. So, the rest of the SSJ machinery of Auclert, Bardóczy, Rognlie and Straub (2021) applies without further modifications. In sum, we're now equipped to solve dynamic general equilibrium models with (or without) rich heterogeneity under general deviations from FIRE.

Corollary 1. *Consider the setup of proposition 2 with FIRE Jacobian $\mathcal{J}$, and forecast-error Jacobian $\mathcal{E}$.*

Let the constant expectations be $dX^e = \Lambda dX$, according to (16). The linearized impulse response dY to an arbitrary shock dX is

$$dY = \left[\left(\mathcal{J} - \mathcal{E} \right) \Lambda + \mathcal{E} \right] dX \tag{18}$$

Corollary 2. *Consider the setup of proposition 3 with FIRE Jacobian $\mathcal{J}$, and forecast-revision Jacobians $\mathcal{R}^\tau$. Let expectations be $dX^{e,t} = \Lambda_t dX$, according to (16). The linearized impulse response dY to an arbitrary shock dX is*

$$dY = \left[\mathcal{J} \Lambda_0 + \sum_{\tau \geq 1} \mathcal{R}_\tau \left(\Lambda_\tau - \Lambda_{\tau-1} \right) \right] dX \tag{19}$$

Given a model for beliefs, the matrices given in equations (18) or (19) fully summarize the response of the heterogeneous agent block to the path $d\mathbf{X}$, taking into account forecast errors and revisions. These matrices are easy and fast to compute after obtaining the FIRE Jacobians $\mathcal{J}$.

4 HANK model with imperfect expectations

The analysis in Section 2 illustrates the importance of expectations in shaping the efficacy of UI policy in terms of output stabilization. However, the stylized nature of that model implies a number of shortcomings in matching a number of important empirical targets. In this section, we generalize the model. Importantly, our model is consistent with a number of details on the micro incidence of unemployment spells and UI expiration; including, the differences of marginal propensities to consume between employed and unemployed workers and income and consumption dynamics though unemployment spells. At the macro level, our model is consistent with evidence on intertemporal multipliers and incentives for job creation. We also model expectations in a general way which nests rational expectations, diagnostic expectations, and incomplete information. This general description of expectations is essential in matching the evidence on the response of expectations in survey data.

The model we propose is a New Keynesian model with heterogeneous households, search and matching unemployment, sticky prices and wages, investment adjustment costs, and smooth fiscal policy (gradual tax adjustments, long-term bonds). We build on models of automatic stabilizers (McKay and Reis, 2016, Kekre, 2021), and medium-scale New Keynesian models (Christiano, Eichenbaum and Trabandt, 2016, Auclert, Rognlie and Straub, 2020, Lee, 2021). Appendix C contains detailed derivations of the equilibrium conditions.

4.1 Households

The household block is a standard incomplete markets model. Households are heterogeneous across four dimensions. First, households have productivity $z_{i,t}$. Second, each household has liquid assets $a_{i,t}$. Third, to model unemployment and UI expiration, we allow households to differ in terms of their employment status $e_{i,t}$, where $e_{i,t} = E$ denotes employment, $e_{i,t} = U$, denotes unemployment receiving UI benefits, and $e_{i,t} = N$ denotes a state of unemployment without

benefits. Finally, we allow households to differ in terms of their discount factor $\beta_i \in \{\beta_1, \beta_2, \beta_3\}$. The mass of households with discount factor β_k is μ_k.

The timing of the shocks within each period is as follows.

1. **Productivity shock.** At the beginning of the period the household draws a new labor productivity, $z_{i,t}$, which follows a Markov process governed by a transition matrix Π_z. We take the discretized Markov process from Kaplan et al. (2018).

2. **Labor market transitions.** First, employed workers lose their job with probability $s_{i,t}$. Second, unemployed workers (including those who separated in this quarter) find jobs with the endogenous probability f_t. Third, newly unemployed workers qualify for unemployment benefits with probability π^{get}, while other households on UI lose eligibility with probability π_t^{lose}. The probability of losing UI eligibility maps directly to the expected duration of benefits $1/\pi_t^{lose}$ and is the key policy variable. The combined transition matrix for labor market status e_{it} is

$$
\begin{array}{c}
\quad\quad\quad E_t \quad\quad\quad\quad\quad U_t \quad\quad\quad\quad\quad N_t \\
\begin{array}{c} E_{t-1} \\ U_{t-1} \\ N_{t-1} \end{array}
\left(
\begin{array}{ccc}
1 - s_i(1 - f_t) & \pi^{get} s_i(1 - f_t) & (1 - \pi^{get}) s_i(1 - f_t) \\
f_t & (1 - \pi_t^{lose})(1 - f_t) & \pi_t^{lose}(1 - f_t) \\
f_t & 0 & 1 - f_t
\end{array}
\right)
\end{array}
\tag{20}
$$

We follow Kekre (2021), and assume that the separation rate is correlated with the discount factor

$$s_i = s + \Delta_\beta(\beta_i - \beta).$$

This feature allows us to match the quantitative differences of wealth and marginal propensities to consume between the employed and unemployed populations.

3. **Consumption-saving decision.** Households choose consumption $c_{i,t}$ and liquid assets $a_{i,t}$ to maximize their expected lifetime utility subject to a budget constraint and a borrowing constraint.

The Bellman equation at the consumption-saving stage is

$$
\begin{aligned}
V_{k,t}(e_{i,t}, z_{i,t}, a_{i,t-1}) = \max_{c_{i,t}, a_{i,t}, h_{i,t}} \; & u(c_{i,t}) + \beta_k E_t[V_{k,t+1}(e_{i,t+1}, z_{i,t+1}, a_{i,t})] \\
\text{s.t. } & c_{i,t} + a_{i,t} = (1 + r_{t-1}^a) a_{i,t-1} + (1 - \tau_t) y_t(e_{i,t}, z_{i,t})^{1-\lambda} \\
& a_{i,t} \geq \underline{a}
\end{aligned}
\tag{21}
$$

where $y_t(e_{i,t}, z_{i,t})$ denotes the household's labor income in employment state $e_{i,t}$ and productivity

state $z_{i,t}$. Following Kekre (2021), we model labor income in the following flexible way:

$$y_t(e_{i,t}, z_{i,t}) = \begin{cases} w_t z_{i,t}, & \text{if } e_{i,t} = E \\ b_1 w_t z_{i,t}, & \text{if } e_{i,t} = U \\ b_2 w_t z_{i,t}, & \text{if } e_{i,t} = N \end{cases}.$$

If the household is employed, they receive their labor income $w_t z_{i,t}$. If the household is unemployment but receiving benefits, they receive a constant replacement rate $b(1 - \omega_0)$ of the persistent component of productivity. This allows us to approximate the dependence of UI benefits on the last wage before unemployment, without introducing more state variables. Furthermore, we also assume that the household receives a fraction ω_1 of their labor income if employed. This feature allows us to model in reduced form the possibility that these households have two earners and is important to match the observed income declines upon unemployment. Finally, if the household stops receiving UI benefits they receive a fraction ω_2 of their income if employed. As in the previous case, ω_2 captures the possibility that the household has a secondary earner. We allow ω_2 to be different from ω_1 to allow for the possibility that the second earner takes active measures to increase labor income in the event of household income loss, as in Bardóczy (2020).

Expectations As in Adam and Marcet (2011), we assume that individuals are internally rational, but may not form correct expectations of variables that are external to them. In this model, these variables include the real interest rate, the tax rate, the aggregate wage, and the job finding probability. In section 4.6, we discuss the specific model of expectations that we impose and the empirical findings that this model allows us to match. For now, it is sufficient to impose three minimal restrictions on expectations.

First, we assume that in the long-run expectations converge to their true correct-steady state values. This assumption means that, while people may make forecast errors throughout the transition, they will not make permanent forecast mistakes. Second, we assume that the law of iterated expectations holds at the individual level, i.e., $E_t[E_{t+j}[\cdot]] = E_t[\cdot]$. This assumption means that individuals do not expect to make forecast errors, even if they actually make systematic forecast errors given the objective probability distribution.[9] These properties are shared by a variety of models of beliefs, including full-information and rational expectations, noisy-information and rational expectations, sticky expectations, diagnostic expectations, or cognitive discounting. Finally, we assume that individuals have the correct expectations of idiosyncratic shocks, which means that they know the correct transition probabilities for idiosyncratic productivity z.

Crucially, the combination of these three assumptions implies that the steady state of our HANK economy coincides exactly with that which is obtained under full-information and rational expectations.

---

[9]This assumption is violated in models where people are sophisticated regarding their future forecast biases, as in Lian (2023).

4.2 Financial intermediary

All assets in the economy are held by a representative financial intermediary. The assets are three: shares in firm equity v_t, long-term nominal government bonds B_t, and short-term nominal reserves M_t. The liabilities of the financial intermediary are net worth N_t^{FI} and short-term deposits A_t from households. Thus the balance sheet, in date-t real terms, is

$$p_t v_t + q_t^B \frac{B_t^N}{P_t} + \frac{M_t}{P_t} = N_t^{FI} + A_t \tag{22}$$

where P_t is the price level, p_t is the equity price, q_t^B is price of long nominal bonds, and the price of reserves is 1. Going forward, let $\pi_t = P_t / P_{t-1} - 1$ denote the inflation rate.

The nominal return on these assets are the following. One share of equity purchased in period $t - 1$ yields dividend stream $\{P_{t+s} d_{t+s}\}$ for all $s \geq 0$. One government bond purchased in period $t - 1$ pays a coupon δ_B^s in period $t + s$ for all $s \geq 0$. One unit of reserves purchased in period $t - 1$ pays $(1 + i_{t-1})$ in period t. Finally, the intermediary pays out d_t^{FI} as dividend to households in period t. This implies that net worth is

$$N_t^{FI} = \underbrace{(d_t + p_t) v_{t-1}}_{\text{gross return on equity}} + \underbrace{\frac{1 + \delta_B q_t^B}{1 + \pi_t} \frac{B_{t-1}}{P_{t-1}} + \frac{1 + i_{t-1}}{1 + \pi_t} \frac{M_{t-1}}{P_{t-1}}}_{\text{gross return on nominal assets}} - \underbrace{(1 + r_{t-1}^a) A_{t-1} - d_t^{FI}}_{\text{pay to households}} \tag{23}$$

For simplicity, we assume that the financial intermediary consumes their dividends.

The financial intermediary is risk neutral. Optimality implies the following asset pricing equations

$$1 + r_t^a = \mathbb{E}_t \left[\frac{d_{t+1} + p_{t+1}}{p_t} \right] = \mathbb{E}_t \left[\frac{1 + \delta_B q_{t+1}^B}{q_t^B (1 + \pi_{t+1})} \right] = \mathbb{E}_t \left[\frac{1 + i_t}{1 + \pi_{t+1}} \right] \equiv 1 + r_t \tag{24}$$

where we defined r_t as the economy-wide ex-ante real interest rate.

4.3 Firms

Our specification of firms is standard. We consider three sectors: retailers (nominal rigidities), capital producer (investment adjustment cost), and labor agency (search and matching frictions). These sectors are connected by competitive markets, so one could model them as one type of firm that makes the same decisions subject to the same constraints.

Retailers. There is unit mass of retailers indexed by j who engage in monopolistic competition. They produce differentiated goods using a Cobb-Douglas production function with the same productivity $y_{jt} = \Theta_t k_{jt}^\alpha l_{jt}^{1-\alpha}$. Firms hire capital k_{jt} and labor l_{jt} on spot markets at prices r_t^k and r_t^l and pay a fixed cost Ξ. They also set the price of their product, p_{jt}, subject to a demand curve with constant elasticity ϵ and a quadratic price adjustment cost à la Rotemberg (1982). We allow for price indexation, so the adjustment cost is paid on price changes relative to a fraction ι_p of last

period's price change. The firms' objective is to maximize the present value of their future profits. The Bellman equation is

$$J_t^R(p_{jt-1}, p_{jt-2}) = \max_{k_{jt}, l_{jt}, y_{jt}, p_{jt}} \left\{ \frac{p_{jt}}{P_t} y_{jt} - r_t^l l_{jt} - r_t^k k_{jt} - \Psi_{jt}^p - \Xi + E_t \left[\frac{J_{t+1}^R(p_{jt}, p_{jt-1})}{1 + r_t} \right] \right\}$$

$$\text{s.t. } y_{jt} = \Theta_t k_{jt}^\alpha n_{jt}^{1-\alpha}$$

$$y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\epsilon} Y_t$$

$$\Psi_{jt}^p = \frac{\psi_p}{2} \left[\log \left(\frac{p_{jt}}{p_{jt-1}} \right) - \iota_p \log \left(\frac{p_{jt-1}}{p_{jt-2}} \right) \right]^2 Y_t$$

In a symmetric equilibrium, all firms choose the same level of output, capital, and labor. So, they have the same marginal cost:

$$mc_t = \frac{1}{\Theta_t} \left(\frac{r_t^k}{\alpha} \right)^\alpha \left(\frac{r_t^l}{1 - \alpha} \right)^{1-\alpha} \tag{25}$$

and set the same prices according to the Phillips curve

$$\pi_t - \iota_p \pi_{t-1} = \frac{\psi_p}{\epsilon} \left(mc_t - \frac{\epsilon - 1}{\epsilon} \right) + \frac{1}{1 + r_t} E_t \left[\frac{Y_{t+1}}{Y_t} \left(\pi_{t+1} - \iota_p \pi_t \right) \right] \tag{26}$$

Capital producer. A representative firm owns the capital stock and rents it to retailers at rate r_t^k. It's Bellman equation is

$$J_t^K(K_{t-1}, I_{t-1}) = \max_{K_t, I_t} \left\{ r_t^k K_{t-1} - I_t + E_t \left[\frac{J_{t+1}^K(K_t, I_t)}{1 + r_t} \right] \right\}$$

$$\text{s.t. } K_t = (1 - \delta)K_{t-1} + \mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) \right] I_t \tag{27}$$

where $\delta \in (0, 1)$ is the depreciation rate, I_t is investment, μ_t is the marginal efficiency of investment as in Justiniano, Primiceri and Tambalotti (2010), and $S(\bullet)$ is a convex function that satisfies $S(1) = S'(1) = 0$.

Defining Tobin's Q as the marginal value of capital at the end of period t, investment dynamics is characterized by

$$Q_t = \frac{r_{t+1}^k + E_t \left[Q_{t+1} (1 - \delta) \right]}{1 + r_t} \tag{28}$$

$$1 = Q_t \mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) - \left(\frac{I_t}{I_{t-1}} \right) S' \left(\frac{I_t}{I_{t-1}} \right) \right] + E_t \left[\frac{\mu_{t+1} Q_{t+1}}{1 + r_t} \left(\frac{I_{t+1}}{I_t} \right)^2 S' \left(\frac{I_{t+1}}{I_t} \right) \right] \tag{29}$$

Labor agency. A representative firm hires workers on a frictional labor market and rents homogeneous labor services to retailers at rate r_t^l. The agency posts vacancies v_t, each of which is

filled with probability q_t. Following Christiano, Eichenbaum and Trabandt (2016), we assume a two-tiered cost of hiring. The firm pays κ_v to create a vacancy and then κ_h for each vacancy it fills.[10] Incumbent workers separate exogenously with probability s_t. The agency takes as given the average hours per worker, H_t. The Bellman equation is

$$J_t^L(N_{t-1}) = \max_{N_t, v_t} \left\{ (r_t^l - w_t) H_t N_t - (\kappa_v + \kappa_h q_t) v_t + E_t \left[\frac{J_{t+1}^L(N_t)}{1 + r_t} \right] \right\} \tag{30}$$

$$\text{s.t. } N_t = (1 - s_t) N_{t-1} + q_t v_t$$

Optimization yields a standard job creation curve, equating the cost and benefit of hiring the marginal worker

$$\frac{\kappa_v}{q_t} + \kappa_h = (r_t^l - w_t) H_t + E_t \left[\frac{1 - s_{t+1}}{1 + r_t} \left(\frac{\kappa}{q_{t+1}} + \kappa_h \right) \right] \tag{31}$$

4.4 Government policy

The fiscal authority issues long-term nominal bonds, collects income taxes, and provides unemployment benefits. Let U_t denote the mass of workers eligible for unemployment benefits. The government budget constraint is

$$G_t + UI_t + \frac{(1 + \delta_B q_t^B)}{1 + \pi_t} \frac{B_{t-1}}{P_{t-1}} = T_t + q_t^B \frac{B_t}{P_t} \tag{32}$$

where UI_t denotes unemployment insurance payments, and T_t denotes tax revenues. Government spending G_t is exogenous. The income tax rate τ_t is chosen according to a rule that can prevent large swings in the tax rate, while ensuring that real government debt is stationary

$$T_t - T_{ss} = \phi_B q_{ss}^B \left(\frac{B_{t-1}}{P_{t-1}} - \frac{B_{ss}}{P_{ss}} \right) \tag{33}$$

In the announcement-based policy, UI duration $1/\pi_t^{lose}$ is exogenous. In the rule-based policy, it is indexed to the end-of-period unemployment rate

$$\frac{1}{\pi_t^{lose}} - \frac{1}{\pi_{ss}^{lose}} = -\zeta_b (N_t - N_{ss}) \tag{34}$$

The monetary authority sets the short-term nominal interest rate according to

$$i_t = i_{ss} + \phi_\pi \pi_t + \epsilon_t^m \tag{35}$$

where ϵ_t^m is a monetary policy shock.

---

[10]The role of κ_h is similar to wage stickiness in models without search and matching. It dampens the procyclicality of marginal costs and hence profits. This is especially important in HANK models where strongly countercyclical profits can have large—and unrealistic—redistributive effects (Broer, Harbo Hansen, Krusell and Öberg, 2020).

4.5 Equilibrium

Wage setting. We assume an ad-hoc wage rule that that ties the evolution of wages to the evolution of the productivity of labor, but delivers realistic real-wage rigidity:

$$\frac{w_t}{w_{ss}} = \left(\frac{h_t}{h_{ss}}\right)^{1-\rho_w} \left(\frac{w_{t-1}}{w_{ss}} - \pi_t\right)^{\rho_w}.$$

Matching. New matches are formed on the labor market according to a Cobb-Douglas matching function

$$M(JS_t, v_t) = A_m(JS_t)^\ell v_t^{1-\ell} \tag{36}$$

where the mass of job seekers equals the mass of unemployed workers from last period plus the mass of newly separated workers

$$JS_t = 1 - N_{t-1} + s_t N_{t-1} \tag{37}$$

Let $\theta_t \equiv v_t / JS_t$ denote labor market tightness. Job finding and vacancy filling probabilities are

$$f_t = A_m \theta_t^{1-\ell} \qquad \text{and} \qquad q_t = \frac{f_t}{\theta_t}. \tag{38}$$

Market clearing. Factor market clearing requires

$$N_t = \int n_{jt} dj \tag{39}$$

$$K_{t-1} = \int k_{jt} dj \tag{40}$$

The notation reflects that capital is predetermined from the perspective of capital producers but not from the perspective of retailers. Aggregate dividends are given by

$$d_t = d_t^R + d_t^K + d_t^L = Y_t - w_t N_t - I_t - \Psi_t^p - (\kappa_v + \kappa_h q_t) v_t - \Xi \tag{41}$$

Firm equity is then priced according to (24). Asset market clearing corresponds to the balance sheet of the financial intermediary (22), imposing that the intermediary holds all shares $v_t = 1$, and nominal reserves are zero $M_t = 0$. Nominal reserves are in zero net supply, the purpose of including them is to deliver a Fisher equation in (24). Goods market clearing requires that the final good is used for household consumption, investment (including adjustment costs), government spending, price adjustment costs, hiring costs, and the fixed cost. Add consumption of financial intermediary.

$$Y_t = C_t + I_t + G_t + \Psi_t^p + (\kappa_v + \kappa_h q_t) v_t + \Xi \tag{42}$$

4.6 Estimation

We estimate the model in two steps, similarly to Christiano, Eichenbaum and Evans (2005) and Auclert, Rognlie and Straub (2020). In the first step, we pin down all the parameters that affect the steady state. We fix some parameters to conventional values from the literature, and calibrate others internally to hit steady-state moments. In the second step, we estimate the remaining parameters by impulse response matching.

Calibration of steady state.   As we discuss above, our assumption on expectations imply that the steady state coincides with the standard FIRE steady state. In other words, deviations from FIRE do not affect the steady state of our economy and thus the computation is standard.

Households have CRRA utility over consumption $u(c) = c^{1-\sigma^{-1}}/(1-\sigma^{-1})$ with an EIS of $\sigma = 0.5$. We set the borrowing limit to $\underline{a} = 0$. We assume that the annual economy-wide real interest rate is $r = 2\%$. Our Markov process for labor productivity $(\mathcal{G}_z, \Pi_z)$ is the discrete-time equivalent of the process estimated by Kaplan, Moll and Violante (2018). We set the income tax progressivity parameter $\lambda = 0.181$ as in Heathcote, Storesletten and Violante (2017). Mean productivity is normalized to 1.

For labor market transitions, we set the job-finding rate to $f = 0.6$. We assume that UI benefits replace 50% of the steady-state wage, and all unemployed workers qualify for benefits initially, $\pi^{get} = 1$. In steady state, unemployment benefits last on average for 2 quarters, $\pi^{lose} = 0.5$. We set the vacancy filling rate to $q = 0.7$ quarterly, and assume that κ_h accounts for 94% of total search cost, leaving 6% for vacancy posting cost per hire κ_v/q. We calibrate the bargaining power of the union η such that total search cost is 7% of the quarterly wage of an average worker.

We calibrate total factor productivity, Θ, to normalize output to $Y = 1$. We set government debt, B/P, to 46% of annual output, and choose the coupon, δ_B, to match the average duration of U.S. government debt of 5 years. Having realistic duration prevents counterfactually large exposure of government budget to fluctuations in short-term interest rates. This matters in non-Ricardian models. We set government spending, G, to 16% of output, which leads to a tax rate of $\tau = 0.29$. We set depreciation rate to $\delta_K = 0.083/4$ quarterly and calibrate the capital share α to match a quarterly capital to output ratio of 8.92. This implies that the steady-state labor share is 62%. The fix cost Ξ is calibrated to make total wealth $p + q_b B/P$ equal to 382% of annual output.

One of the most important transition-specific parameters is ζ_b, the semi-elasticity of average unemployment duration with respect to the unemployment rate. We pin this down from a linear approximation of the Emergency Unemployment Compensation (EUC08) program. Our goal is to work with a policy rule that is in the right ballpark, not to provide a serious quantitative evaluation of EUC08 per se.[11] The unemployment rate in 2007Q1 was 4.6%, close to the steady state value of 4.5% in our model. Unemployment rate peaked at 10.1% in 2009Q4. During the same time, unemployment benefit duration was raised from 26 weeks to 99 weeks (in states with

---

[11]EUC08 features nonlinearities and a staggered rollout which we ignore. Kekre (2021) for example takes these features into account but assumes perfect foresight with respect to the announced policy.

unemployment rate above 8.5%). So, our back of the envelope calculation for the semi-elasticity is $\zeta_b = (99 - 26)/13/(0.101 - 0.046) \approx 102$. That is, a one percentage point increase in the unemployment rate triggers 1.02 quarter increase in average UI duration.

Table 1: Targeted Moments

Moment	Data	Model
Mean quarterly MPC	0.21	0.21
Annual MPC unemp. - emp.	0.25	0.25
Unemployment rate	0.05	0.05
UI recipient share	0.39	0.39
HH income w UI / pre job loss	0.76	0.76
HH income after UI / pre job loss	0.55	0.55

This leaves us seven parameters to be calibrated internally. The average discount factor β_2, the dispersion in discount factors $\beta_3 - \beta_2 = \beta_2 - \beta_1$, the average separation rate s, the elasticity of the separation rate with respect to the discount factor Δ_β, and the probability of qualifying for unemployment benefits upon job loss π^{get}, and the replacement rates b_1 and b_2. We calibrate these seven parameters to target five moments on table 1, which we take from Kekre (2021). The average quarterly MPC and the difference between the average MPC of unemployed and employed workers are informative about the discount factors and the elasticity of separation rates to discount factors. The unemployment rate pins down the average separation rate. The UI recipient share pins down the probability of qualifying for unemployment benefits. Finally, the income loss upon job loss and after benefit exhaustion pin down the replacement rates.

Table 2: Untargeted Moments

Moment	Data	Model
HH consumption w UI / pre job loss	0.92	0.91
HH consumption after UI / pre job loss	0.76	0.76
Mean liquid net worth / monthly HH income	3.7	4.25
Liquid net worth unemp. - emp.	-2.6	-1.78
Aggregate consumption shares by net worth		
Quintile 5 (highest)	0.37	0.42
Quintile 4	0.22	0.19
Quintile 3	0.17	0.15
Quintile 2	0.12	0.13
Quintile 1 (lowest)	0.11	0.08

The seven parameters span the space of moments we target, thus we can hit the targeted moments exactly. Table 2 shows that the model does remarkably well on key untargeted moments as well. Crucially, the model matches almost perfectly the average consumption drop upon job loss

and upon benefit exhaustion. The amount of wealth the model requires to match the MPC targets conditional on the income and unemployment risk is close to the liquid net worth of households in the data. Despite allowing for impatient households to be disproportionately exposed to unemployment risk, the model still understates the wealth difference between employed and unemployed households. Finally, the model does a good job matching joint distribution of consumption and wealth.

Estimation: IRF Matching. As in Christiano, Eichenbaum and Evans (2005), we estimate the model by matching the impulse response functions obtained in our model to their empirical counterparts obtained with a standard business-cycle shock. We generate the empirical impulse responses by following the empirical strategy in Angeletos, Huo and Sastry (2021). That is, we estimate the regression

$$z_t = \alpha + \sum_{p=1}^{P} \gamma_p z_{t-p}^{IV} + \sum_{k=0}^{K} \beta_k \epsilon_{t-k} + u_t \tag{43}$$

where z_t is an outcome of interest (e.g. unemployment rate), ϵ_t is an identified shock, and z_{t-p}^{IV} are lagged values of z_t instrumented by lagged values of ϵ_t. Our identified shock is the main business cycle (MBC) shock from Angeletos, Collard and Dellas (2020). This shock is constructed to account for most of the business cycle fluctuations in unemployment rate. We generate impulse response functions not only for outcomes, but also generate the impulse response functions of expectation for the relevant variables at several horizons.

To perturb the economy from its steady-state level, we consider a single shock to the marginal efficiency of investment (MEI) which follows an AR(1) process

$$\mu_t = \rho_\mu \mu_{t-1}.$$

The process for this shock has two free parameters: the persistence ρ_μ and the initial level of the shock μ_0. Angeletos, Collard and Dellas (2020) show that the MBC and MEI shocks are closely related. Using our model, we solve the impulse responses of variables and expectations in our model to this shock for a given set of parameters.

We recover the implied impulse response functions IRF(Ω). Ω denotes the set of that we estimate which can be seen in Table 4. We choose values for these parameters so as to minimize the distance between our model's implied impulse response and those estimated in the data:

$$\hat{\Omega} = \arg \min_{\Omega} \left(\text{IRF}(\Omega) - \widehat{\text{IRF}} \right)' \Sigma^{-1} \left(\text{IRF}(\Omega) - \widehat{\text{IRF}} \right),$$

where $\widehat{\text{IRF}}$ denotes the estimated impulse response function. In our estimation, we include the impulse response functions for the unemployment rate, consumption, inflation rate, and the nominal interest rate.

To solve our economy, we use the Sequence-Space Jacobian method for general expectations

24

in equation (15). This method allows us to flexibly parameterize expectations. We now describe how we choose the model of expectations to match salient features of expectations in survey data.

Expectations In practice, the exercise described in the previous section cannot be fully implemented due to the unavailability of all necessary expectations data. In this dimension, we confront two limitations: (1) we may not have survey data on expectations to all relevant variables and (2) even for the variables for which we do have survey data for, we only observe expectations for a finite number of future horizons, and not the infinite number of horizons which would be required to solve the model.

We use data for the Survey of Professional Forecasters (SPF). From this dataset, we use data for one to four quarters-ahead unemployment rate forecasts.[12] However, people must still form expectations about the real-interest rate, tax rates, job-finding rates, among others. Furthermore, they must also form expectations about the unemployment rate at horizons beyond the fourth quarter ahead. We solve both of these issues by imposing a parametric model of beliefs to generate beliefs of variables for which expectations data are lacking and extrapolate unemployment forecasts beyond the fourth quarter horizon.

As Angeletos, Huo and Sastry (2021) point out, most popular models of belief formation generate either under-reaction or over-reaction at all horizons. This pattern is very clearly seen in the impulse response of forecasts observed in Figure 1. To capture the estimated pattern of initial under-reaction followed by delayed over-reaction, we combine noisy information with diagnostic expectations and long memory. In doing so, we build on Bordalo et al. (2020) (who combined noisy information with standard diagnostic expectations) and on Bianchi et al. (2021) (who introduced diagnostic expectations with long memory). In Appendix D.1, we discuss the merits of this model of beliefs relative to other popular models in the literature. As Figure D.1 shows, having both features is essential to match the estimated pattern.

As we discuss in Appendix B.3, the noisy-information and long-memory diagnostic expectations model implies that the time t average expectation to a deterministic shock takes the following form:

$$\overline{E}_t[dX_{t+h}] = \underbrace{\left[(1+\theta)\frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1} - \theta \sum_{j=1}^{t} \alpha_j \left(\frac{t+1-j}{\tau_\epsilon/\tau_\nu + t + 1 - j}\right)\right]}_{\equiv \lambda_t} dX_{t+h}, \tag{44}$$

where $\overline{E}_t[X_{t+h}]$ denotes the average expectation and $\mathbb{E}_t[X_{t+h}]$ denotes the full-information and rational expectations in that same economy, and uses the following convention

$$\sum_{j=1}^{t} \alpha_j \left(\frac{t+1-j}{\tau_\epsilon/\tau_\nu + t + 1 - j}\right) = 0$$

if $t = 0$. This model features several parameters: θ denotes the degree of belief over-reaction,

---

[12]We are currently working on incorporating SPF data for other variables into our framework.

$\alpha_j \geq 0$ for $j \geq 1$ denote the memory weights and satisfy $\sum_{j=1}^{\infty} \alpha_j = 1$, and $\tau \equiv \tau_\epsilon / \tau_\nu$ denotes the ration of the precision of priors to the precision of the noisy signals.

Table 3: Belief parameters

Parameter	Description	Value
θ	Diagnostic expectation param	4.332
τ	Noisy information param	10.304
α	Long memory param 1	7.536
β	Long memory param 2	24.907

This model nests four known models as special cases. First, assume that $\tau = 0$ and $\alpha_1 = 1$. Then, this model collapses to the standard diagnostic expectations, as in Bordalo, Gennaioli and Shleifer (2018). Second, maintaining the assumption that $\tau = 0$ but allowing for the memory weights to assign mass to further away expectations, our model also nests the long-memory diagnostic expectation model used in Bianchi, Ilut and Saijo (2021). Third, assuming that $\theta = 0$ but $\tau > 0$, this model collapses to the standard noisy-information and rational expectations model as in Angeletos and Huo (2021). Finally, allowing $\theta > 0$ and $\tau > 0$ but assuming that $\alpha_1 = 1$, then this model collapses to the standard noisy-information and diagnostic expectations model used in Bordalo, Gennaioli, Ma and Shleifer (2020). Our model is best understood as extending this final model to allow for long-memory, which turns out to be essential in capturing the pattern of initial under-reaction followed by delayed over-reaction which can be seen in Figure 1.[13]

As in Bianchi, Ilut and Saijo (2021), we assume that the α_j are determined by a Beta-binomial distribution with parameters α and β. This assumption implies that we have four parameters to calibrate in this model θ, τ, α, and β. We calibrate these parameters so that the beliefs that they would imply for the unemployment rate forecasts line up with those that we observe in the data. However, note that, in solving the model, we actually use directly the observed unemployment rate forecasts and not the ones implied by this model.

The calibrated parameters are found in Table 3 and the models empirical match can be seen in Figure 2. Overall, the fit to the data we actually observe is good. Note that, in this model, the ratio of under or over-reaction,

$$\frac{\overline{E}_t[dX_{t+h}]}{dX_{t+h}} = \lambda_t,$$

is constant across horizons. As it turns out, to match the data, the implied forecasts slightly exagerate the amount of over-reaction at the shortest horizon while underestimating the amount of over-reaction at longer horizon. We leave a more in depth analysis of this interesting fact for future work.

---

[13]See Appendix D.1 for a discussion.

Figure 2: Calibration of belief parameters
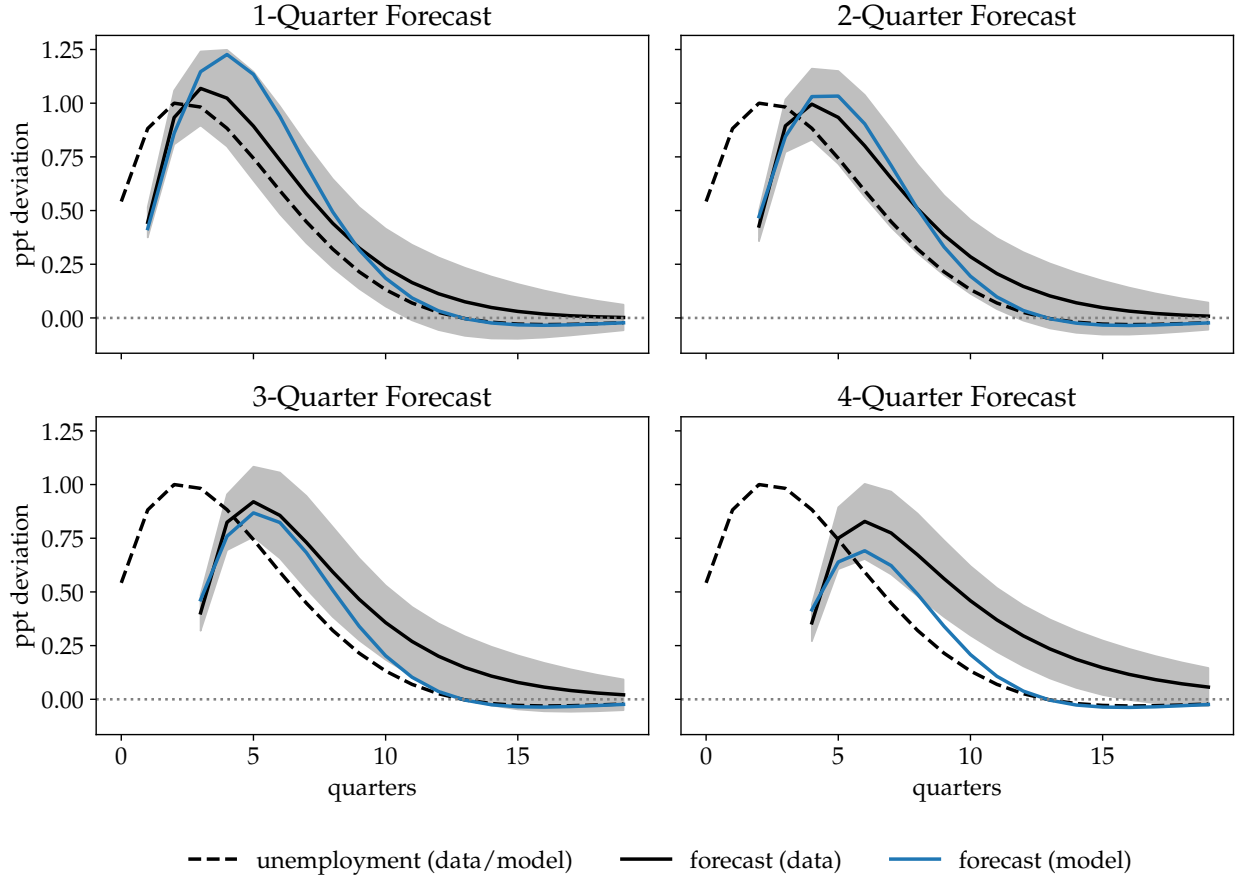
5 Results

In this section, we present the main results of our estimation exercise. Furthermore, we compare the implied impulse responses in our model to the benchmarks of perfect and no anticipation of future changes. This exercise allows us to understand the implications of the patterns of imperfect expectations observed in the data.

5.1 Estimation results

We estimate the remaining parameters which are relevant for the transition dynamics in our economy. These parameters are as follows. The monetary policy parameters: the Taylor-rule coefficient on inflation, ϕ_π, the Taylor-rule coefficient on unemployment, ϕ_u, and the Taylor-rule inertia, ρ_m. The investment adjustment cost, ψ, and the real-wage adjustment cost, ψ_w. The elasticity of the tax rate to debt, ϕ_B. The nominal rigidity parameters: price indexation, ι_p, and the slope of the Phillips curve, κ_p. The financial income payout rate, ϕ_N. Finally, the parameters controlling the

scale and the persistence of the MEI shock: μ_0 and ρ_μ, respectively. The estimated values for the parameters which affect the transition dynamics in our economy can be found in Table 4.

Table 4: Estimated parameters

Parameter	Description	Value
Core model parameters		
ϕ_π	Taylor rule coef on inflation	1.241
ϕ_u	Taylor rule coef on unemployment	0.122
ρ_m	Taylor rule inertia	0.000
ψ	Investment adjustment cost	1.788
ϕ_B	Response of tax rate to debt	0.054
ψ_w	Real wage adjustment cost	1082.0
ι_p	Price indexation	0.249
κ_p	Phillips curve slope	0.075
ϕ_N	Financial income payout rate	0.009
MEI shock process		
μ_0	Scale of MEI shock	0.025
ρ_μ	Persistence of MEI shock	0.716

The model's impulse response functions are shown in the blue lines in Figure 3 for the unemployment rate, consumption, inflation rate, and the nominal interest rate. The black line shows the associated empirical impulse responses the the shaded region plots the 68% confidence interval around the empirical point estimates. The model provides a good fit to its targeted empirical counterparts.

Figure 3: Targeted Impulse Responses: Outcomes
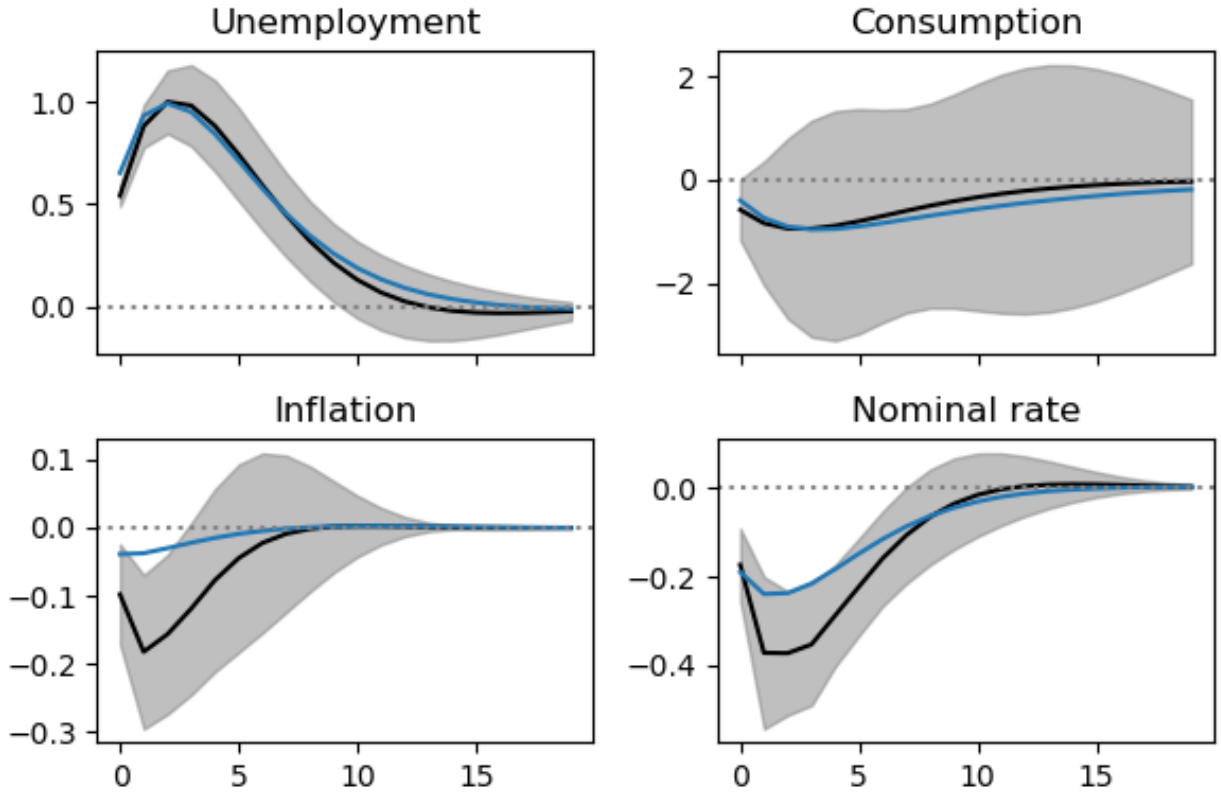
## 5.2 Quantifying the consequences of imperfect expectations

In this section, we assess the efficacy of UI extensions on stimulating demand with imperfect expectations. The goal is to quantify the role of imperfect anticipation of endogenous UI extensions in affecting aggregate demand. As we have discussed before, we do so by working directly with the empirical response of expectations, avoiding the need to choose a particular model of belief formation. We show that the direct effect of UI extensions on the distribution of income is less important than their indirect effect on precautionary saving. This implies that the power of UI extensions to boost aggregate demand is diminished if households do not anticipate them. We show this result in partial equilibrium (using only the calibrated household block) as well as in general equilibrium (using the full estimated HANK model).

Partial-equilibrium analysis. UI extensions can boost aggregate demand by two channels. First, directly, by raising the income of unemployed households who get to keep their benefits thanks to the extension. Second, indirectly, by reducing the precautionary savings of employed households facing the risk of job loss, and of unemployed households facing the risk of losing benefits. Our

29

first goal is to establish that the precautionary saving channel is quantitatively relevant.

We consider a UI extension that would be triggered, according to policy rule (34), by the empirical impulse response of unemployment with respect to the main business cycle shock. The path of UI duration is plotted in the left panel of Figure 4. We feed this path of UI duration to the households of our HANK model and compute the response of aggregate consumption under different assumptions about expectations. For the purposes of this partial equilibrium exercise, we keep all other prices, income, and the job-finding rate constant at their steady-state level.[14] We contrast the response of the economy under the estimated beliefs with two extreme benchmarks: (1) full-information and rational expectations (FIRE) and (2) myopia. The first benchmark assumes that people have the correct expectations, which implies that they make no forecast errors. The second benchmark assumes that people never revise their beliefs about the future and so consistently make forecast errors. So the first benchmark features perfect anticipation of UI benefits, while the second benchmark features no anticipation of UI benefits.

Figure 4: Partial-equilibrium Consumption Responses to an UI Extension
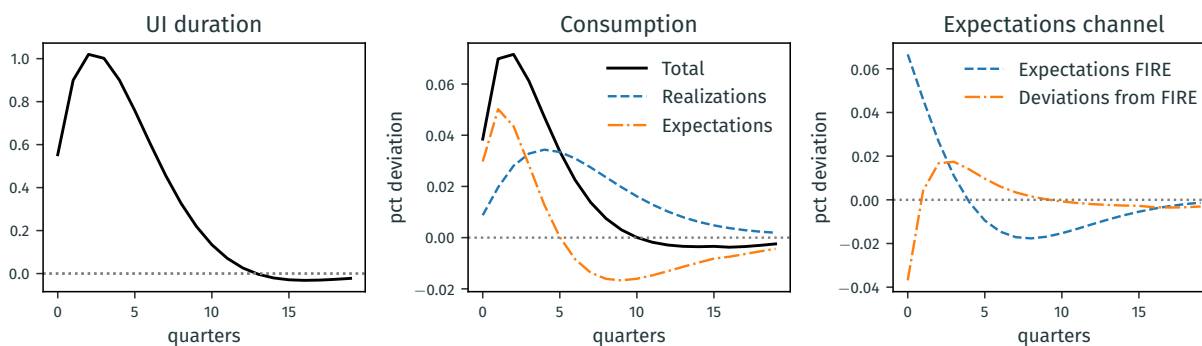
- Very important role for anticipation – Expectations explain 63% of first year response.

  – Expectations are important – precautionary savings channel

- Deviations from FIRE have a substantial impact

  – reduce initial response by 2/3, but increase response in second year

The right panel of Figure 4 shows the impulse response of aggregate consumption. The blue line is computed assuming that households have FIRE or perfect foresight of the rise in UI duration from period 0 onwards. In this scenario, aggregate consumption rises sharply on impact due to an immediate reduction in precautionary savings, stays above steady state for five quarters, and then falls below steady state as households start to build back their normal buffer stock of savings. To understand why consumption falls below steady state before it recovers, note that a UI extension

---

[14]That is, our results depend only on the calibrated household block, and are independent from the supply side and policy blocks of the model.

raises incomes only for those workers that lose their job and stay eligible longer. The majority of households remain continuously employed during the period of the UI extension. From their perspective, UI extension reduces risk, but provides no income. They optimally adjust their buffer stock in response to the change in unemployment risk.

The second scenario, myopia, isolates the role of actual transfers, as household (wrongly) forecast no change to their income prospects upon unemployment. The orange line shows that the resulting response of aggregate consumption is markedly different from the first scenario with FIRE. Conditional on their individual states, the households' consumption-saving decisions do not change at all. The hump-shaped aggregate consumption response is driven entirely by changes in the distribution. The mass of UI eligible households rises while the mass of ineligible unemployed households falls. Aggregate consumption rises moderately because the households who receive UI benefits consume more on average than those who exhausted their benefits. The comparison of this scenario with FIRE demonstrates the importance of anticipation of UI benefits in shaping the consumption response to the policy. In fact, the peak response of aggregate consumption to unemployment benefits is over four times as large with FIRE than with myopia, and it happens on impact as opposed to 5 quarters later.

In the third scenario, we give households the expectations estimated in the ARMA-IV regression (43). As figure 1 shows, estimated beliefs feature initial dampening followed by delayed over-reaction relative to the actual path of the unemployment rate. Since we assume that households have first-order knowledge of the policy, the same pattern applies to beliefs about UI duration. It follows that the initial response of aggregate consumption is muted relative to FIRE, but much higher than the myopic scenario since people still anticipate some of the UI extension. However, because of the over-reaction in beliefs, after a few periods the response of aggregate demand becomes even higher than under FIRE due to the effect of perceived UI duration on precautionary savings.

Figure 5: Cumulative Spending out of UI Transfers
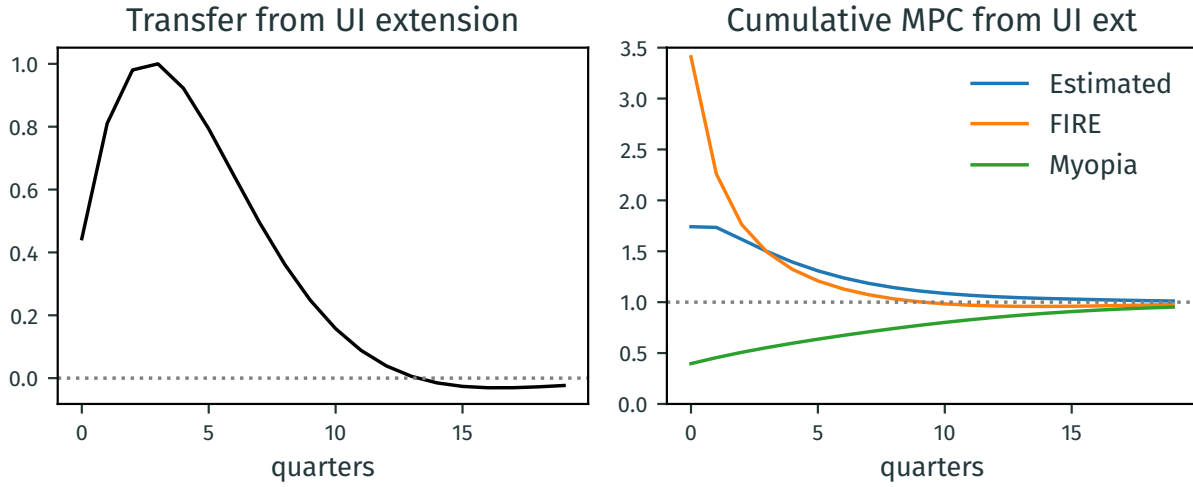
General-equilibrium analysis. We established that imperfect anticipation of UI extensions has a large impact on the partial-equilibrium response of aggregate demand to the policy. Next, we compute the consequences imperfect anticipation in the full dynamic general-equilibrium model.

Figure 6 displays the impulse response of aggregate consumption to the marginal efficiency of investment (MEI) shock. As in the previous section, we compare the response in our baseline economy with the estimated beliefs to the benchmarks of perfect anticipation (FIRE) and no anticipation (Myopia). In performing these comparisons, we fix all parameters (other than those relating to beliefs) to their estimated values (see section 4.6). We then compute the dynamic response of those benchmarks to the same MEI shock. The response of our baseline economy can be seen in green, while the response under FIRE and Myopia can be seen in blue and orange.

Figure 6: General Equilibrium Impulse Responses to MEI Shock
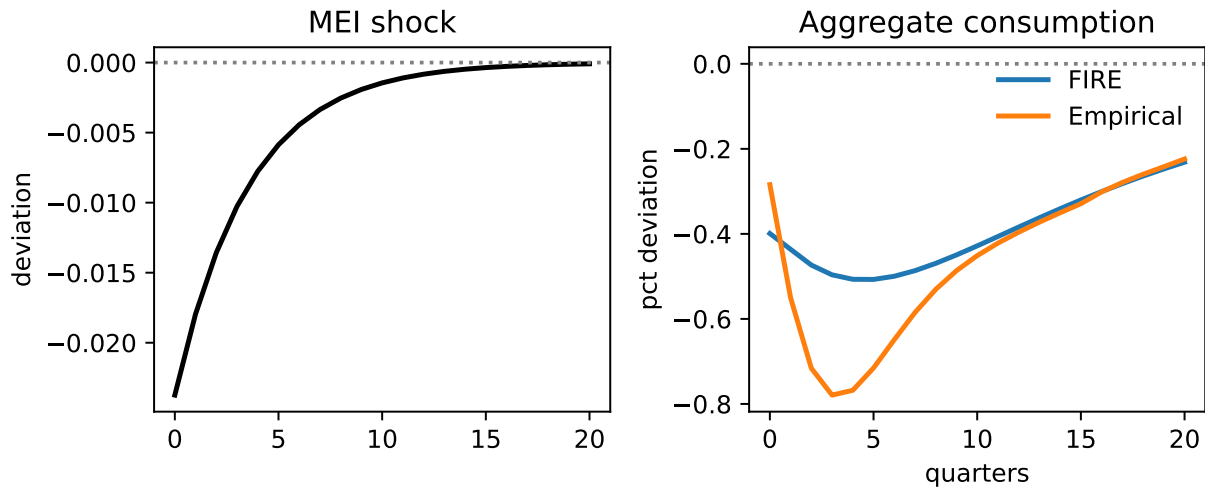
Aggregate consumption falls in response to this negative MEI shock for all models, mostly because firms invest less and hire less workers, leading to a rise in unemployment and a decline in incomes. As unemployment surges the government responds by increasing UI benefits, which helps stimulate the economy, but does not fully offset the shock.

The initial drop in consumption is less pronounced in the baseline model than with FIRE. This result is a consequence of the fact that individuals are more optimistic about the depth of the recession due to the initial under-reaction of beliefs, i.e., individuals think that unemployment will not rise as much. The same holds for the comparison of Myopia to the two other lines. The initial drop in consumption is -0.39, -0.58, and -0.16 percent for the baseline, FIRE, and Myopia economies.

However, after this initial period, individuals become more pessimistic about the future path of unemployment and job finding prospects than they would under FIRE or Myopia. It follows that individuals predict larger unemployment risk and so, despite also predicting higher UI benefits, they have a higher precautionary-savings motive and cut their consumption by more relative to FIRE and Myopia. These effects imply a hump-shaped response of aggregate consumption which would not be present with FIRE. The peak response of aggregate consumption with the estimated beliefs if -0.81, while for FIRE it is equal to the initial response -0.58. Over time, these very pessimistic expectations are not realized and individuals consume their excess savings, justifying the fact that consumption is higher after 10 quarters under the estimated beliefs than under both other benchmarks.

Figure 6 highlights the importance of expectations on the general-equilibrium response of aggregate consumption. However, it does not allow us to understand the independent effects of each general-equilibrium channel. To better understand the consequences of the fall in job finding rates and the endogenous rise in UI duration, we now decompose the overall GE effect. We isolate

the effect of different channels on consumption, by taking the Jacobian with respect to that input and multiplying it by the impulse response of that input. We focus primarily on understanding the effects coming through these two channels due to their central importance in our analysis. In appendix D.2, we complement the analysis here by describing the effects of the remaining GE forces.

6 Quantifying the stimulative power of UI extensions

The results in the previous section allow us to understand the importance of anticipating UI extensions in determining their efficacy in stimulating consumption. Furthermore, they allow us to compare the implications of the empirical patters of beliefs relative to two important benchmarks: FIRE and myopia. However, those results do not allow us to quantify the power of UI extensions. For that purpose, we need to compare the impulse responses obtained in the previous section with those obtained in a counterfactual economy assuming no extensions, i.e., $\zeta_b = 0$.

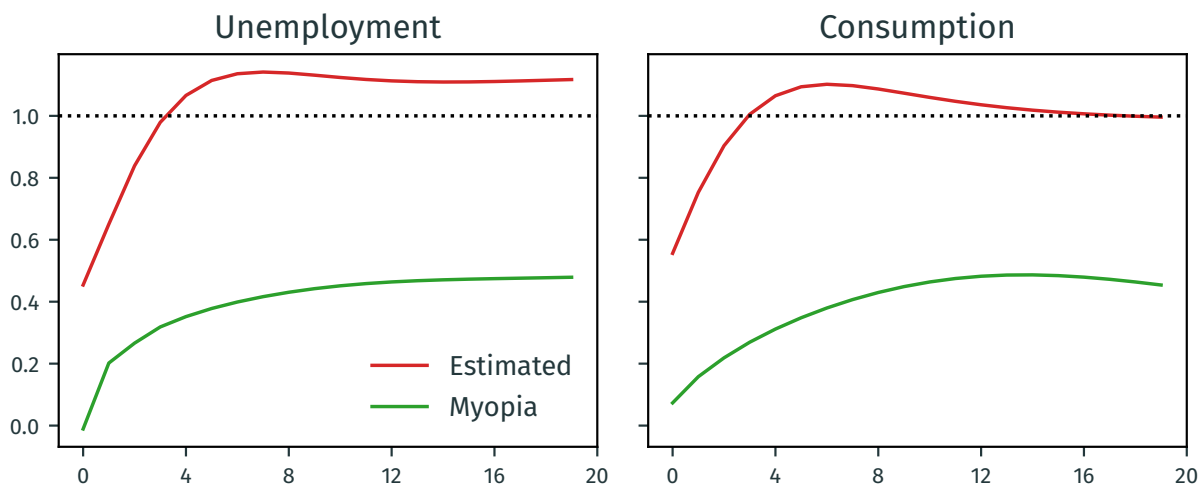Figure 7: Cumulative Efficacy of Countercyclical UI relative to FIRE

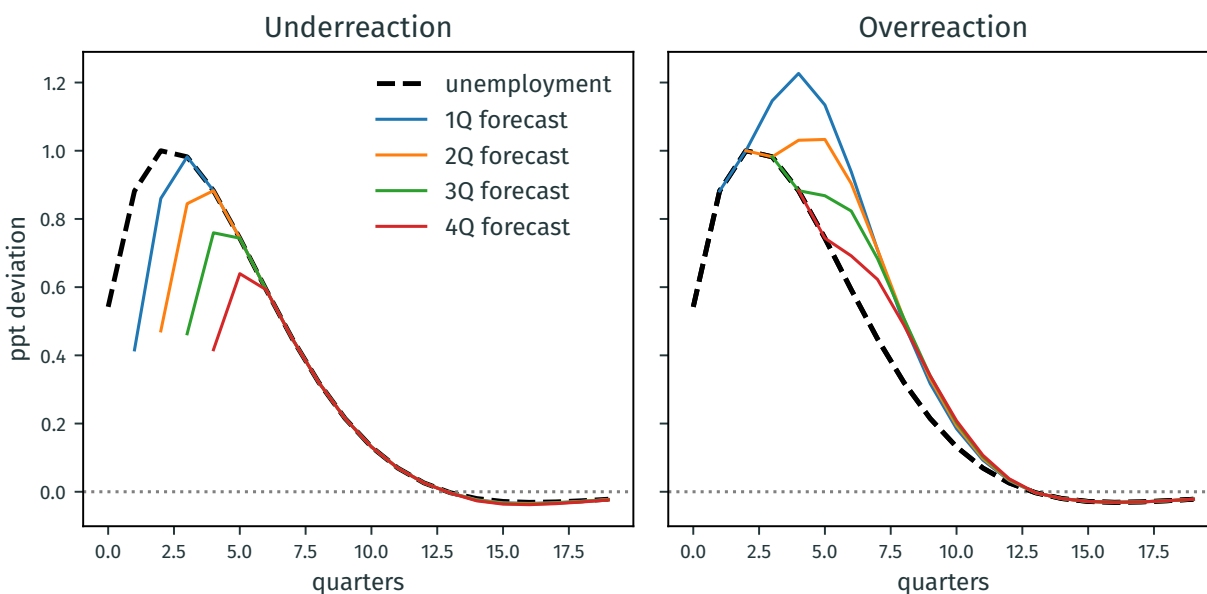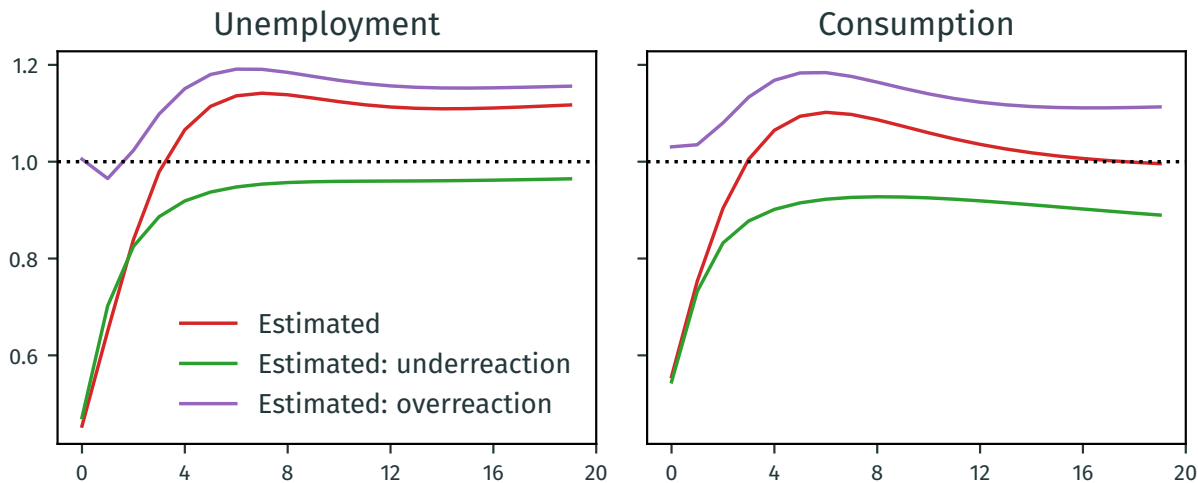Figure 8: Under- vs. Over-reaction of expectations

Figure 9: Cumulative Efficacy of Countercyclical UI relative to FIRE: Under- vs. Over-reaction

The role of under- vs. over-reaction of expectations

7 Alternative policy implementation

In this section, we are interested in understanding how the efficacy of UI duration extensions is affected by the way in which the policy is implemented. As in Bianchi-Vimercati, Eichenbaum and

Guerreiro (2021), we are interested in comparing the impact of the policy when it is implemented and announced as a rule versus when the path of UI duration is directly announced. In the latter case, we assume that the government directly announces a path for the policy variable π_t^{lose} and that this announcement is immediately learned and understood by all market participants.

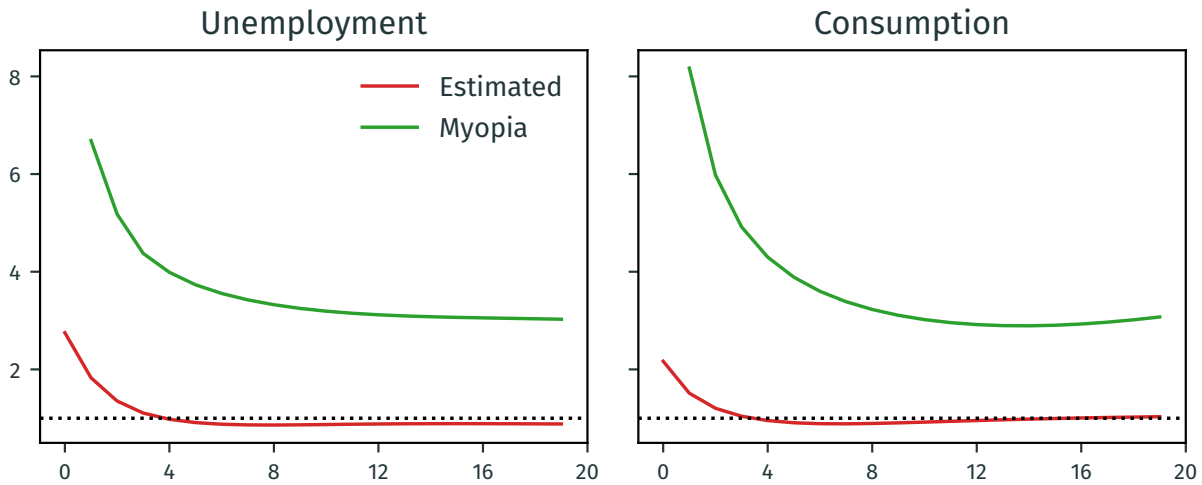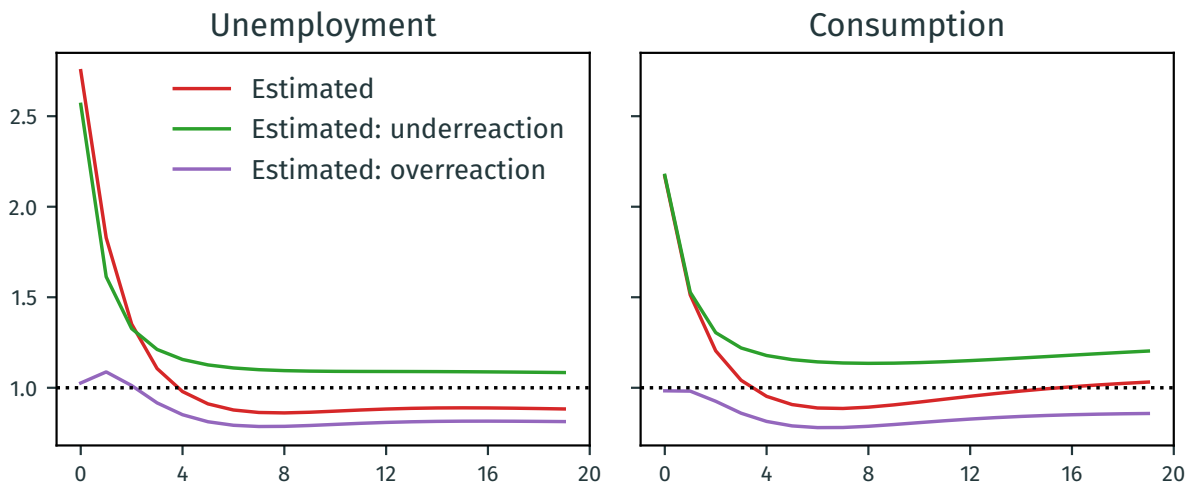Figure 10: Cumulative Efficacy of Countercyclical UI relative to FIRE

Figure 11: Cumulative Efficacy of Countercyclical UI relative to FIRE

The role of under- vs. over-reaction of expectations   As in Section 6, making this comparison requires us to compute a counterfactual path for the economy under a different policy implementation. So in our baseline economy, we maintain the assumption that beliefs are given by the

noisy-information and long-memory diagnostic expectations model (baseline) that we estimate in Section 4.6. We first compute the response of the economy under the assumption that the policy is implemented as a rule. We then recover the implied path for UI duration and compute the dynamic response in the counterfactual economy where the same policy is directly announced at the onset of the recession. As in the previous section, we do this analysis in our baseline economy and for comparison also perform the analysis under FIRE and myopia.

Figures **??**, **??**, and **??** display the impulse responses under FIRE, myopia, and baseline beliefs, respectively. For each figure, the left panel displays the impulse responses for the unemployment rate with the rules-based policy (blue) and with the instrument-announcement policy (brown). The black dashed line plots the difference between the equilibrium with announcement and that with rules. The right panel displays the analogous three responses for consumption.

Figure **??** shows that with perfect anticipation, i.e., with FIRE, the two forms of policy implementation lead to the same outcomes. With FIRE, it doesn't make any difference whether the instruments are directly announced or that they are announced as a rule, since people accurately forecast the behavior of the unemployment rate and can thus use it to perfectly predict the future path of unemployment duration (see also Angeletos and Sastry, 2021, and Bianchi-Vimercati, Eichenbaum and Guerreiro, 2021). Instead, Figure **??** shows that with myopia there can be large differences between these two forms of implementation. When the instrument is directly announced to people, the model features large anticipation of UI benefits and so a strong stimulative power of the policy. Indeed, the unemployment rate falls by over 0.5 percentage points on impact and consumption is boosted by almost 0.7 percentage points on impact.

Finally, in our baseline economy, the results are mixed. The announcement-based policy limits the initial rise in the unemployment rate by over 0.25 percentage points, and the fall in consumption by 0.28 percentage points. After the initial period, the order is reversed and the unemployment rate becomes 0.15 percentage points higher in the economy with the announcement-based policy. Similarly, consumption falls 0.25 percentage points more. These results are a direct consequence of the pattern of delayed over-reaction present in beliefs. Initially, beliefs under-react, so people under-forecast the generosity of UI extensions in the economy with the policy rule. So it is more powerful to directly announce the extension in UI benefits.[15] However, after this initial period, beliefs over-react and people become overly pessimistic. So under the rule-based policy, people believe that UI benefits will be extended for longer. In other words, the rule-based policy exploits the belief over-reaction and stimulates demand further without an actual increase in UI duration.

We conclude that in our baseline economy, changing the implementation of UI extensions to a direct announcement could help their stimulative power at the onset of the recession, but may lack efficacy past the peak of the recession when beliefs turn overly pessimistic.

---

[15]This argument follows the same logic as in Bianchi-Vimercati et al. (2021).

8 Conclusion

Economists have long emphasized the benefits of linking UI benefits duration to aggregate economic conditions (see, e.g., Chodorow-Reich and Coglianese 2019, Eichenbaum 2019, Mitchell and Husak 2021). In this paper, we argue that expectations are critical in determining the stabilization power of these policies.

We study the economic impact of UI extensions in a state-of-the-art Heterogeneous Agent New Keynesian model with search and matching frictions. We discuss a general framework to solve and analyze such models under arbitrary beliefs about macroeconomic outcomes. We leverage the framework to estimate the model to match the impulse responses of key aggregate variables and expectations to identified business-cycle shocks. By doing so, we demonstrate that expectations data can be used directly to solve the model, thus sidestepping the issue of choosing among the "wilderness" of alternative models for belief formation. Our results emphasize that the stimulative power of state-dependent UI extensions can be greatly affected by systematic forecast errors that people make in predicting the business cycle.

References

Adam, Klaus and Albert Marcet, "Internal Rationality, Imperfect Market Knowledge and Asset Prices," *Journal of Economic Theory*, 2011, *146* (3), 1224–1252.

Angeletos, George-Marios and Karthik A Sastry, "Managing Expectations: Instruments Versus Targets," *The Quarterly Journal of Economics*, 2021, *136* (4), 2467–2532.

__ **and Zhen Huo**, "Myopia and Anchoring," *American Economic Review*, 2021, *111* (4), 1166–1200.

__ **, Fabrice Collard, and Harris Dellas**, "Business-Cycle Anatomy," *American Economic Review*, 2020, *110* (10), 3030–70.

__ **, Zhen Huo, and Karthik A Sastry**, "Imperfect Macroeconomic Expectations: Evidence and Theory," *NBER Macroeconomics Annual*, 2021, *35* (1), 1–86.

Auclert, Adrien, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub, "Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models," *Econometrica*, 2021, *89* (5), 2375–2408.

__ **, Matthew Rognlie, and Ludwig Straub**, "Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model," Working Paper 26647, National Bureau of Economic Research 2020.

Bardóczy, Bence, "Spousal Insurance and the Amplification of Business Cycles," Job Market Paper, Northwestern University 2020.

Bianchi, Francesco, Cosmin L Ilut, and Hikaru Saijo, "Diagnostic Business Cycles," Working Paper 28604, National Bureau of Economic Research 2021.

Bianchi-Vimercati, Riccardo, Martin S Eichenbaum, and Joao Guerreiro, "Fiscal Policy at the Zero Lower Bound without Rational Expectations," Working Paper 29134, National Bureau of Economic Research August 2021.

Bilbiie, Florin, Giorgio E Primiceri, and Andrea Tambalotti, "Inequality and Business Cycles," Manuscript 2022.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer, "Diagnostic Expectations and Credit Cycles," *The Journal of Finance*, 2018, *73* (1), 199–227.

__ , __ , **Yueran Ma, and Andrei Shleifer**, "Overreaction in Macroeconomic Expectations," *American Economic Review*, 2020, *110* (9), 2748–82.

Broer, Tobias, Niels-Jakob Harbo Hansen, Per Krusell, and Erik Öberg, "The New Keynesian Transmission Mechanism: A Heterogeneous-Agent Perspective," *The Review of Economic Studies*, 2020, *87* (1), 77–101.

Cagan, Phillip, "The Monetary Dynamics of Hyperinflation," *Studies in the Quantity Theory of Money*, 1956.

Carroll, Christopher D, "The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence," *Brookings Papers on Economic Activity*, 1992, *1992* (2), 61–156.

__ , "Macroeconomic Expectations of Households and Professional Forecasters," *Quarterly Journal of Economics*, 2003, *118* (1), 269–298.

__ , **Edmund Crawley, Jiri Slacalek, Kiichi Tokuoka, and Matthew N White**, "Sticky Expectations and Consumption Dynamics," Working Paper 224377, National Bureau of Economic Research 2018.

Chodorow-Reich, Gabriel and John M Coglianese, "Unemployment Insurance and Macroeconomic Stabilization," *Unemployment Insurance and Macroeconomic Stabilization." In Recession Ready, ed. Heather Boushey, Ryan Nunn, and Jay Shambaugh*, 2019.

Christiano, Lawrence J, Martin Eichenbaum, and Charles L Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 2005, *113* (1), 1–45.

__ , **Martin S Eichenbaum, and Mathias Trabandt**, "Unemployment and Business Cycles," *Econometrica*, 2016, *84* (4), 1523–1569.

Coibion, Olivier and Yuriy Gorodnichenko, "What Can Survey Forecasts Tell us About Information Rigidities?," *Journal of Political Economy*, 2012, *120* (1), 116–159.

__ **and** __ , "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts," *American Economic Review*, 2015, *105* (8), 2644–78.

Dobrew, Michael, Rafael Gerke, Sebastian Giesen, and Joost Röttger, "Make-Up Strategies with Incomplete Markets and Bounded Rationality," Bundesbank Discussion Paper 2023.

Eichenbaum, Martin S, "Rethinking Fiscal Policy in an Era of Low Interest Rates," *Economic Policy Group Monetary Authority of Singapore*, 2019, *90.*

Farhi, Emmanuel and Iván Werning, "Monetary policy, Bounded Rationality, and Incomplete Markets," *American Economic Review*, 2019, *109* (11), 3887–3928.

__ **, Mikel C. Petri, and Iván Werning**, "The Fiscal Multiplier Puzzle: Liquidity Traps, Bounded Rationality, and Incomplete Markets," Technical Report, mimeo 2020.

Fernandes, Adriano and Rodolfo Rigato, "Precautionary Savings and Stabilization Policy in a Present-Biased Economy," 2022. Job market paper.

Friedman, Milton, "A Theory of the Consumption Function," in "A Theory of the Consumption Function," Princeton University Press, 1957, pp. 1–6.

Gabaix, Xavier, "A Sparsity-Based Model of Bounded Rationality," *The Quarterly Journal of Economics*, 2014, *129* (4), 1661–1710.

__ , "Behavioral Macroeconomics via Sparse Dynamic Programming," Technical Report, National Bureau of Economic Research 2016.

__ , "A Behavioral New Keynesian Model," *American Economic Review*, 2020, *110* (8), 2271–2327.

Guerreiro, Joao, "Belief Disagreement and Business Cycles," Job Market Paper, Northwestern University 2022.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L Violante, "Optimal Tax Progressivity: An Analytical Framework," *The Quarterly Journal of Economics*, 2017, *132* (4), 1693–1754.

Justiniano, Alejandro, Giorgio E Primiceri, and Andrea Tambalotti, "Investment Shocks and Business Cycles," *Journal of Monetary Economics*, 2010, *57* (2), 132–145.

Kaplan, Greg, Benjamin Moll, and Giovanni L Violante, "Monetary Policy According to HANK," *American Economic Review*, 2018, *108* (3), 697–743.

Kekre, Rohan, "Unemployment Insurance in Macroeconomic Stabilization," Working Paper 29505, National Bureau of Economic Research 2021.

Kimball, Miles S., "Precautionary Saving in the Small and in the Large," *Econometrica*, 1990, *58* (1), 53–73.

Krusell, Per, Toshihiko Mukoyama, and Ayşegül Şahin, "Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations," *Review of Economic Studies*, 2010, *77* (4), 1477–1507.

Lee, Donggyu, "The Effects of Monetary Policy on Consumption and Inequality," Technical Report, Working paper 2021.

Lian, Chen, "Mistakes in Future Consumption, High MPCs Now," *American Economic Review: Insights*, 2023, *5* (4), 563–581.

Mankiw, N Gregory and Ricardo Reis, "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," *The Quarterly Journal of Economics*, 2002, *117* (4), 1295–1328.

McKay, Alisdair and Ricardo Reis, "The Role of Automatic Stabilizers in the US Business Cycle," *Econometrica*, 2016, *84* (1), 141–194.

Mitchell, David and Corey Husak, "How to Replace COVID Relief Deadlines with Automatic 'Triggers' that Meet the Needs of the US Economy," *The Washington Center for Equitable Growth*, 2021.

Mitman, Kurt and Stanislav Rabinovich, "Optimal Unemployment Insurance in an Equilibrium Business-Cycle Model," *Journal of Monetary Economics*, 2015, *71*, 99–118.

_ **and** _ , "Do Unemployment Benefit Extensions Explain the Emergence of Jobless Recoveries?," CEPR Discussion Paper No. DP13760 2019.

Nakajima, Makoto, "A Quantitative Analysis of Unemployment Benefit Extensions," *Journal of Monetary Economics*, 2012, *59* (7), 686–702.

Pappa, Evi, Morten O. Ravn, and Vincent Sterk, "Expectations and Incomplete Markets," in "Handbook of Economic Expectations," Elsevier, 2023, pp. 569–611.

Rotemberg, Julio J, "Sticky Prices in the United States," *Journal of Political Economy*, 1982, *90* (6), 1187–1211.

Sargent, Thomas J, *The Conquest of American Inflation*, Princeton University Press, 2001.

Sims, Christopher A, "Macroeconomics and Reality," *Econometrica: journal of the Econometric Society*, 1980, pp. 1–48.

_ , "Implications of Rational Inattention," *Journal of Monetary Economics*, 2003, *50* (3), 665–690.

Werning, Iván, "Incomplete Markets and Aggregate Demand," Working Paper 21448, National Bureau of Economic Research 2015.

Woodford, Michael, "Imperfect Common Knowledge and the Effects of Monetary Policy," Working Paper 8673, National Bureau of Economic Research December 2001.

_ , "Monetary Policy Analysis when Planning Horizons are Finite," Working Paper 24692, National Bureau of Economic Research June 2018.

A Appendix to section 2

A.1 Deriving the equilibrium

We solve the model backwards, starting at the end of period 1. At this point, households observe the relevant equilibrium outcomes $\{b_1, \tau_1\}$ as well as their employment status $e_1 \in \{E, U\}$. Their problem is

$$V_1(e_1, a_0) = \max_{c_1, a_1} u(c_1) \quad \text{s.t. } c_1 + a_1 = (1 + r)a_0 + \mathbf{1}_{\{e_t = E\}} + \mathbf{1}_{\{e_1 = U\}} \cdot b_1 - \tau_1$$

$$a_1 \geq 0 \tag{A.1}$$

where we already imposed $w_1 = 1$. Clearly, the optimal decision is to consume the entire cash on hand

$$a_1(e_1, a_0) = 0 \tag{A.2}$$

$$c_1(e_1, a_0) = (1 + r)a_0 + \mathbf{1}_{\{e_1 = E\}} + \mathbf{1}_{\{e_1 = U\}} \cdot b_1 - \tau_1 \tag{A.3}$$

Since assets are in zero net supply and borrowing is not allowed, all workers have zero assets in equilibrium, $a_0 = 0$. Given the policies $\{b_1, M_1\}$, the time-1 equilibrium can be computed recursively as

$$N_1 = M_1 \tag{A.4}$$

$$\tau_1 = (1 - N_1)b_1 \tag{A.5}$$

$$c_1(E) = 1 - \tau_1 \tag{A.6}$$

$$c_1(U) = b_1 - \tau_1 \tag{A.7}$$

Note that time-1 equilibrium is independent of what happens in period 0, including the beliefs $\{N_1^e, b_1^e, \tau_1^e\}$ that households hold in period 0.

Let's turn to period 0. Combining the consumption policy function (A.3) with the fact that the probability of employment is iid, the expected continuation value at the end of period 0 can be written as

$$V_1^e(a_0) = N_1^e \cdot u\left((1 + r)a_0 + 1 - \tau_1^e\right) + (1 - N_1^e) \cdot u\left((1 + r)a_0 + b_1 - \tau_1\right) \tag{A.8}$$

At time $t = 0$, households solve

$$\max_{c_0, a_0} u(c_0) + \beta V_1^e(a_0) \quad \text{s.t. } c_0 + a_0 = \mathbf{1}_{\{e_0 = E\}} + \mathbf{1}_{\{e_0 = U\}} \cdot b_0 - \tau_0$$

$$a_0 \geq 0 \tag{A.9}$$

Taking FOCs yields the Euler equation (7) in the main text

$$u'(c_0) \geq \beta(V_1^e)'(a_0) \tag{A.10}$$

$$= \beta(1+r)\left[N_1^e \cdot u'\left(1 - \tau_1^e + (1+r)a_0\right) + (1 - N_1^e) \cdot u'\left(b_1^e - \tau_1^e + (1+r)a_0\right)\right] \tag{A.11}$$

Note that the right-hand side does not depend on time-0 employment status. Since there can't be any saving in equilibrium ($a_0 = 0$), households consume their income in period 0, which is higher for employed households. Given that $u'(\bullet)$ is decreasing, this implies that either both types of households are borrowing constrained or only unemployed workers are constrained. We assume that β and r is such that only unemployed households are constrained.

Then, given beliefs $\{N_1^e, \tau_1^e, b_1^e\}$ and policies $\{b_0, r\}$, the time-0 equilibrium can be computed recursively as follows

$$u'\left(c_0(E)\right) = \beta(1+r)\left[N_1^e \cdot u'\left(1 - \tau_1^e\right) + (1 - N_1^e) \cdot u'\left(b_1^e - \tau_1^e\right)\right] \tag{A.12}$$

$$\tau_0 = 1 - c_0(E) \tag{A.13}$$

$$c_0(U) = b_0 - \tau_0 \tag{A.14}$$

$$N_0 = 1 - \frac{\tau_0}{b_0} \tag{A.15}$$

$$M_0 = N_0 \tag{A.16}$$

A.2 Proof of proposition 1

Proof. Consider an infinitesimal shock to nominal GDP in period 1, dM_1. Differentiating the time-1 equilibrium (A.12)–(A.16) yields

$$dN_1 = dM_1 \tag{A.17}$$

$$d\tau_1 = (1 - N_1)db_1 - dN_1 \cdot b_1 \tag{A.18}$$

$$dc_1(E) = -d\tau_1 \tag{A.19}$$

$$dc_1^U = db_1 - d\tau_1 \tag{A.20}$$

Since we assumed that both UI extension regimes implement the same benefits db_1, the time-1 responses are the same under both regimes. Given our model of beliefs (**??**), this implies that expectations of employment and taxes are the same

$$dN_1^{e,rule} = dN_1^{e,*} = \lambda \cdot dN_1 \quad \text{and} \quad d\tau_1^{e,rule} = d\tau_1^{e,*} = \lambda \cdot d\tau_1 \tag{A.21}$$

but the expectation of unemployment benefits may differ

$$db_1^{e,rule} = -\zeta_b \cdot dN_1^{e,rule} = -\zeta_b \lambda \cdot dN_1 \leq -\zeta_b \cdot dN_1 = db_1^{e,*} \tag{A.22}$$

44

These expectations are relevant for pinning down $dc_0(E)$ through the Euler equation of employed workers (A.12). To first order after the shock, the Euler equation reads as

$$u''(1 - \tau_0) \cdot dc_0(E) = \beta(1 + r)\Big[dN_1^e \cdot u'\Big(1 - \tau_1^e\Big) - N_1^e \cdot u''\Big(1 - \tau_1^e\Big)d\tau_1^e$$
$$- dN_1^e u'\Big(b_1^e - \tau_1^e\Big) + (1 - N_1^e) \cdot u''\Big(b_1^e - \tau_1^e\Big)(db_1^e - d\tau_1^e)\Big] \tag{A.23}$$

So the difference in consumption under the two UI extension regimes is

$$dc_0(E)^{\text{rule}} - dc_0(E)^{\text{ann}} = \underbrace{\frac{\beta(1 + r)(1 - N_1^e) \cdot u''(b_1^e - \tau_1^e)}{u''(1 - \tau_0)}}_{\equiv M_b} \cdot \Big(db_1^{e,\text{rule}} - db_1^{e,*}\Big) \tag{A.24}$$

where $M_b \in [0, 1]$ can be interpreted as the marginal propensity to consume out of anticipated UI benefits. Note that the dN_1^e and $d\tau_1^e$ terms cancel because these expectations are independent of the UI extension regime.

Differentiating the rest of the time-0 equilibrium conditions (A.13)–(A.16) gives us

$$d\tau_0 = -dc_0(E) \tag{A.25}$$
$$dc_0(U) = db_0 - d\tau_0 \tag{A.26}$$
$$dN_0 = -\frac{d\tau_0}{b_0} + \frac{\tau_0}{b_0^2}db_0 \tag{A.27}$$
$$dM_0 = dN_0 \tag{A.28}$$

Let's assume that UI benefits respond only in period 1, $db_0 = 0$, in order to isolate the impact of precautionary behavior. Combining the perturbed time-0 equilibrium conditions proves the proposition

$$dY_0^{\text{rule}} - dY_0^{\text{ann}} = \frac{1}{b_0} \cdot M_b \cdot (1 - \lambda) \cdot dM_1 \tag{A.29}$$

To interpret the $1/b_0$ term, note that the aggregate consumption function of this economy is

$$C_0 = N_0 \cdot c_0(E) + (1 - N_0)(b_0 - \tau_0) \tag{A.30}$$

According to (A.12), the consumption choice of employed workers $c_0(E)$ does not depend on N_0. Also recall that, in equilibrium, $c_0(E) = 1 - \tau_t$. So

$$\frac{\partial C_0}{\partial N_0} = c_0(E) - (b_0 - \tau_0) = 1 - b_0$$

This implies that

$$\frac{1}{b_0} = \frac{1}{1 - \frac{\partial C_0}{\partial N_0}} \equiv \mathcal{M} > 0 \tag{A.31}$$

is a standard Keynesian multiplier. $\qquad\square$

B Appendix to section 3

B.1 Proof of Proposition 2

We consider a generic representation of a heterogeneous-agent problem. Following Auclert et al. (2021), we can write this representation as a mapping between aggregate inputs X_t, a time path for aggregate outputs Y_t. In order to allow for generic beliefs, we extend their framework by allowing X_t^e to denote the expectations that thhe individuals hold about variable X. As we will se below, the behavior of individuals is determined by their forward looking expectations X_t^e and the current realization of the variable X_t. The dimension of X is $n_x \times 1$ and the dimension of Y_t is $n_y \times 1$. We assume that the distribution is discretized on n_g points, i.e., D_t is the $n_g \times 1$ matrix summarizing the distribution of agents. Let $y_t = y\left(v_{t+1}^e, X_t\right)$ be the $n_g \times n_y$ matrix of individual outcomes, given the inputs to the decision problem $\left(v_{t+1}^e, X_t\right)$. The heterogeneous-agent problem can be written as follows:

$$v_t = v\left(v_{t+1}^e, X_t\right) \tag{B.1}$$

$$v_t^e = v\left(v_{t+1}^e, X_t^e\right) \tag{B.2}$$

$$D_{t+1} = \Lambda\left(v_{t+1}^e, X_t\right)' D_t \tag{B.3}$$

$$Y_t = y\left(v_{t+1}^e, X_t\right)' D_t \tag{B.4}$$

We write (Y, v, v^e, D) as the steady state for their respective variables when expectations are correct $X_t^e = X_t = X$. This immediately implies that, in steady state, $v = v^e$. For notational convenience, let $\Lambda \equiv \Lambda\left(v^e, X\right)$ denote the steady state transition matrix. In what follows, we consider transitions of time length T that satisfy that the inputs and expectations converge back to steady state after T periods, i.e., $X_t^e = X_t = X$ for $t \geq T$, which also implies that $v_t^e = v_t = v$ for $t \geq T$. We assume that the heterogeneous-agent block starts from its steady state so $D_0 = D$.

This heterogeneous-agent problem defines a $T \times n_y$ vector of stacked outputs

$$\boldsymbol{Y} = h\left(\boldsymbol{X}, \boldsymbol{X}^e\right),$$

where $\boldsymbol{Y} = \begin{bmatrix} Y_0 & Y_1 & \dots \end{bmatrix}'$, $\boldsymbol{X} = \begin{bmatrix} X_0 & X_1 & \dots \end{bmatrix}'$, and $\boldsymbol{X}^e = \begin{bmatrix} X_0^e & X_1^e & \dots \end{bmatrix}'$. Assuming that all functions are differentiable in $\boldsymbol{X}$ and $\boldsymbol{X}^e$, then h is also differentiable. Our goal is to characterize the Jacobian $\mathcal{J}$ of h with respect to variables X and X^e, evaluated at the steady state. To do this, we consider the problem starting at steady state and perturb in turn the time s input by dX_s or the expectation of the time s input dX_s^e. Importantly, this procedure allows us to disentagle the consequences of changes in expectations from changes in realizations for that variable.

Responde to dX_s   Consider a change to input X at time s, dX_s, with $dX_t = 0$ for all $t \neq s$ and $dX_t^e = 0$ for all t. It follows immediately that

$$v_t^e = v^e = v,$$

for all t, and $v_t = v$ for all $t \neq s$. Furthermore, it follows that, for all $t \neq s$, $y_t \equiv y \left(v_{t+1}^e, X_t \right) = y(v, X) = y$ and $\Lambda_t \equiv \Lambda \left(v_{t+1}^e, X_t \right) = \Lambda(v, X) = \Lambda$, so $dy_t = 0$ and $d\Lambda_t = 0$.

Using the chain rule, we can decompose the change in output at time t by the change in choices y_t and the change in the distribution D_t:

$$dY_t = dy_t' D + y dD_t$$

and

$$dD_{t+1} = d\Lambda_t' D + \Lambda' dD_t.$$

Using these expressions and the results above, it follows that, prior to date s, the output and distribution remain unchanged, i.e., $dY_t = 0$ and $dD_{t+1} = 0$ for all $t < s$. Furthermore, for $t = s$, we find that $dD_s = 0$, so

$$dD_{s+1} = d\Lambda_s' D, \quad \text{and} \quad dY_s = dy_s' D,$$

After date s, there is no other change in behavior, so we only need to account for the change in the distribution, i.e., for $t > s$ we find that

$$dY_t = y dD_t$$

where

$$dD_t = \left(\Lambda' \right)^{t-(s+1)} dD_{s+1}.$$

Finally, note that dy_s does not depend on the time s. It immediately follows that

$$\frac{\partial Y_t}{\partial X_s} = \begin{cases} 0 & \text{if } t < s \\ \frac{\partial Y_{t-s}}{\partial X_0}, & \text{if } t \geq s \end{cases} = \begin{cases} 0 & \text{if } t < s \\ \mathcal{J}_{t-s,0}, & \text{if } t \geq s, \end{cases} \tag{B.5}$$

where $\mathcal{J}$ denotes the FIRE Jacobian derived in Auclert et al. (2021).

Response to dX_s^e   We now consider the response of the heterogeneous-agent output to a shock to expectations that is not realized, i.e., $dX_s^e \neq 0$ but $dX_s = 0$.

Note that, at and after date s, the value functions, transition matrices, and decisions return to their steady state values, i.e., $v_t^e = v_t = v$, $\Lambda_t = \Lambda$ and $y_t = y$, for all $t \geq s$. It follows that, for $t > s$,

$$dY_t = y dD_t$$

and

$$dD_{t+1} = \Lambda' dD_t$$

47

where $dD_t = \Lambda' dD_{t-1} = (\Lambda')^{t-(s+1)} dD_{s+1}$.

It follows that, prior to date s ($t < s$), the response is exactly the same as that which would be obtained under FIRE, i.e., $dY_t = \mathcal{J}_{t,s}$. For $t = s$, we find that $v_{s+1}^e = v$ and since $X_s = X$, then $y_s = y$ and $\Lambda_s = \Lambda$. Let the superscript $*$ denote the variables in the case both the variable and the expectation had changed by the same ammount, i.e., $dX_s = dX_s^e$ which corresponds to the FIRE response. It follows that

$$dD_{s+1} = \Lambda dD_s = \Lambda dD_s^* = dD_{s+1}^* - (d\Lambda_s^*)' D$$

and

$$dY_s = \boldsymbol{y} dD_s^* = dY_s^* - (d\boldsymbol{y}_s^*)' D,$$

where dD_s^* and $d\Lambda_s^*$ denote the response under FIRE, i.e., $dX_s^e = dX_s$.

Let dD_s^0 and $d\Lambda_s^0$ denote the responses to an unanticipated change, i.e., $dX_s \neq 0$ with $dX_s^e = 0$ as we computed above. Then, we can also write

$$dD_{s+1} = \Lambda dD_s = \Lambda dD_s^* = dD_{s+1}^* - (d\Lambda_s^*)' D = dD_{s+1}^* - dD_{s+1}^0$$

and

$$dY_s = \boldsymbol{y} dD_s^* = dY_s^* - (d\boldsymbol{y}_s^*)' D = dY_s^* - dY_s^0,$$

Finally, for $t > s$, we also find that $v_{t+1}^e = v$ and $X_t = \boldsymbol{X}$ so that decisions and transitions do not change. As a result,

$$dD_t = (\Lambda')^{t-(s+1)} dD_{s+1} = (\Lambda')^{t-(s+1)} (dD_{s+1}^* - dD_{s+1}^0) = dD_t^* - dD_t^0$$

$$dY_t = \boldsymbol{y} dD_t = \boldsymbol{y} dD_t^* - \boldsymbol{y} dD_t^0 = dY_t^* - dY_t^0.$$

These results mean that we can decompose an expectations shocks as follows. Prior to date s, the response is exactly identical to what would be obtained under a perfect foresight (FIRE) transition do a shock dX_s, since the only effects come from anticipation. From date s on, we can decompose the response to an (unrealized) expectations shock as the difference between the FIRE response subtracted by an unanticipated time s shock of the exact reverse sign. It follows that we can write:

$$\frac{dY_t}{dX_s^e} = \begin{cases} \mathcal{J}_{t,s} & \text{if } t < s \\ \mathcal{J}_{t,s} - \mathcal{J}_{t-s,0} & \text{if } t \geq s \end{cases} \tag{B.6}$$

Putting it together Define

$$\mathcal{E} \equiv \begin{bmatrix} \mathcal{J}_{0,0} & 0 & 0 & \cdots \\ \mathcal{J}_{1,0} & \mathcal{J}_{0,0} & 0 & \cdots \\ \mathcal{J}_{2,0} & \mathcal{J}_{1,0} & \mathcal{J}_{0,0} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}, \tag{B.7}$$

then we can summarize these results in the following expressions

$$dY = (\mathcal{J} - \mathcal{E}) \cdot dX^e + \mathcal{E} \cdot dX = \mathcal{J} \cdot dX^e + \mathcal{E} \cdot \underbrace{(dX - dX^e)}_{\text{Forecast Error}}. \tag{B.8}$$

B.2 Proof of proposition 3

Let $\left\{ \left\{ X_s^{e,t} \right\}_{s=t+1}^{T-1} \right\}_{t=0}^{T-1}$ denote their beliefs at each point in time. We can extend the previous representation as follows:

$$v_s^{e,t} = v \left(v_{s+1}^{e,t}, X_s^{e,t} \right), \quad s = 0, ..., T-1, t = 0, ..., T-1 \tag{B.9}$$

$$v_t = v \left(v_{t+1}^{e,t}, X_t \right) \tag{B.10}$$

$$D_{t+1} = \Lambda \left(v_{t+1}^{e,t}, X_t \right)' D_t \tag{B.11}$$

$$Y_t = y \left(v_{t+1}^{e,t}, X_t \right)' D_t, \tag{B.12}$$

where $v_{T+1}^{e,t} = v$ and $X_{T+1}^{e,t} = X$ for all t. We still denote the full-information and rational expectations jacobian by $\mathcal{J}$.

The response to a completely unanticipated date s shock dX_s is still the same as in the previous case with time-invariant expectations

$$\frac{dY_t}{dX_s} = \begin{cases} 0 & \text{if } t < s \\ \mathcal{J}_{t-s,0} & \text{if } t \geq s \end{cases}$$

So, we need only extend the previous analysis for the expectations. The main observation is that, when considering a change in time τ expectations about time s input, $dX_s^{e,\tau}$, there are no changes for $t < \tau$, i.e., the problem remains at its steady state level. It follows that the response of output at time t to a change in expectations at time τ about the input at time s is exactly the same as the response of output at time $t - \tau$ to a change in expectations at time 0 about the input at time $s - \tau$:

$$\frac{\partial Y_t}{\partial X_s^{e,\tau}} = \frac{\partial Y_{t-\tau}}{\partial X_{s-\tau}^{e,0}}. \tag{B.13}$$

Using this fact, we need only consider changes in time 0 expectations.

Note that if we shock the time-0 expectations for time $s > 0$, $dX_s^{e,0}$, then $v_s^{e,t} = v$, $y_t = y$, $\Lambda_t = \Lambda$ for $t > 0$, i.e., only time 0 decisions change. What happens at time 0? Let the superscript $*$ denote the variables response for a perfect foresight (FIRE) shock $dX_s = dX_s^{e,t}$ for all t, then:

$$dY_0 = dy_0' D = (dy_0^*)' D$$

$$dD_1 = (d\Lambda_0^*)' D = dD_1^*$$

49

For $t > 0$, since decisions are unchange, then

$$dY_t = y'dD_t$$

$$dD_{t+1} = \Lambda'dD_t = (\Lambda')^t dD_1 = (\Lambda')^t dD_1^*$$

Note that dY_0 is exactly the same as under rational expectations.

Following Auclert et al. (2021), the perfect foresigh (FIRE) responses are given by

$$dD_t^* = (d\Lambda_{t-1}^*)' D + \Lambda'dD_{t-1}^* = \sum_{m=0}^{t-2} (\Lambda')^m (d\Lambda_{t-1-m}^*)' D + (\Lambda')^{t-1} dD_1^*$$

$$dY_t^* = (dy_t^*)' D + y'dD_t^*$$

It is useful to put superscripts for the time of the shock, s. For example, define the FIRE response as:

$$dD_t^{*,s} = \sum_{m=0}^{t-2} (\Lambda')^m (d\Lambda_{t-1-m}^{*,s})' D + (\Lambda')^{t-1} dD_1^{*,s}$$

$$dY_t^{*,s} = (dy_t^{*,s})' D + y'dD_t^{*,s}$$

So, the response to the time-0 expectations shock for the time s input is given by:

$$dD_t^s = dD_t^{*,s} - \sum_{m=0}^{t-2} (\Lambda')^m (d\Lambda_{t-1-m}^{*,s})' D$$

$$dY_t = dY_t^* - (dy_t^{*,s})' D + y' (dD_t^s - dD_t^{*,s}).$$

Note furthermore, that

$$\begin{aligned}
dD_t^{*,s-1} &= \left(d\Lambda_{t-1}^{*,s-1}\right)' D + \Lambda'dD_{t-1}^{*,s-1} \\
&= \left(d\Lambda_{t-1}^{*,s-1}\right)' D + \Lambda' \left(d\Lambda_{t-1}^{*,s-1}\right)' D + (\Lambda')^2 dD_{t-2}^{*,s-1} \\
&= \sum_{m=0}^{t-1} (\Lambda')^m \left(d\Lambda_{t-1-m}^{*,s-1}\right)' D + (\Lambda')^t \underbrace{dD_0^{*,s-1}}_{=0} \\
&= \sum_{m=0}^{t-1} (\Lambda')^m \left(d\Lambda_{t-1-m}^{*,s-1}\right)' D
\end{aligned}$$

and now using the fact that

$$d\Lambda_{t-1-m}^{*,s-1} = d\Lambda_{t-m}^{*,s}$$

we can write

$$dD_t^{*,s-1} = \sum_{m=0}^{t-1} (\Lambda')^m (d\Lambda_{t-m}^{*,s})' D \quad \text{and} \quad dD_{t-1}^{*,s-1} = \sum_{m=0}^{t-2} (\Lambda')^m (d\Lambda_{t-1-m}^{*,s})' D$$

It follows that:

$$dD_t^s = dD_t^{*,s} - dD_{t-1}^{*,s-1}.$$

Finally, we can write

$$dY_t^s = y'dD_t^s = \underbrace{y'dD_t^{*,s}}_{=dY_t^{s,*}-(dy_t^{*,s})'D} -y'dD_{t-1}^{*,s-1} = dY_t^{s,*} - (dy_t^{*,s})'D - y'dD_{t-1}^{*,s-1}$$

and since $dy_t^{*,s} = dy_{t-1}^{*,s-1}$ then

$$dY_t^s = dY_t^{s,*} - \left(\left(dy_{t-1}^{*,s-1}\right)' D + y'dD_{t-1}^{*,s-1} \right) = dY_t^{s,*} - dY_{t-1}^{s-1,*}.$$

It thus follows that

$$\frac{\partial Y_t}{\partial X_s^{e,0}} = \mathcal{J}_{t,s} - \mathcal{J}_{t-1,s-1}. \tag{B.14}$$

Putting everything together   We have thus found that

$$\frac{\partial Y_t}{\partial X_s} = \begin{cases} 0 & \text{if } t < s \\ \mathcal{J}_{t-s,0} & \text{if } t \geq s \end{cases}$$

and

$$\frac{\partial Y_t}{\partial X_s^{e,\tau}} = \begin{cases} 0 & \text{if } t < \tau \text{ or } s \leq \tau \\ \mathcal{J}_{t-\tau,s-\tau} - \mathcal{J}_{t-\tau-1,s-\tau-1} & \text{if } t > \tau \text{ and } s > \tau \\ \mathcal{J}_{0,s-t} & \text{if } t = \tau \text{ and } s > \tau = t. \end{cases}$$

Putting everything together we can write

$$dY_t = \sum_{s=0}^{t} \mathcal{J}_{t-s,0} \cdot dX_s + \sum_{\tau=0}^{t-1} \sum_{s=\tau+1}^{\infty} \left(\mathcal{J}_{t-\tau,s-\tau} - \mathcal{J}_{t-\tau-1,s-\tau-1} \right) \cdot dX_s^{e,\tau} + \sum_{s=t+1}^{\infty} \mathcal{J}_{0,s-t} \cdot dX_s^{e,t} \tag{B.15}$$

$$= \sum_{\tau=1}^{t} \sum_{s=\tau}^{\infty} \mathcal{J}_{t-\tau,s-\tau} \left(dX_s^{e,\tau} - dX_s^{e,\tau-1} \right) + \sum_{s=0}^{\infty} \mathcal{J}_{t,s} dX_s^{e,0} \tag{B.16}$$

from where equation (15) follows immediately.

B.3   Special cases

Throughout, we maintain the following notation. $E_t[dX_{t+h}]$ denotes the agent's time-t expectations about the variable at horizon h. $\mathbb{E}_t[dX_{t+h}]$ denotes the full-information and rational expectation for the same variable.

B.3.1 Shallow reasoning

(Angeletos and Sastry, 2021). $E_t[dX_{t+h}] = \lambda \cdot dX_{t+h}$.

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & \lambda & 0 & 0 & \dots \\ 0 & 0 & \lambda & 0 & \dots \\ 0 & 0 & 0 & \lambda & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & \lambda & 0 & \dots \\ 0 & 0 & 0 & \lambda & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.17}$$

B.3.2 Cognitive discounting

(Gabaix, 2020). $E_t[dX_{t+h}] = \lambda^h \cdot dX_{t+h}$.

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & \lambda & 0 & 0 & \dots \\ 0 & 0 & \lambda^2 & 0 & \dots \\ 0 & 0 & 0 & \lambda^3 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & \lambda & 0 & \dots \\ 0 & 0 & 0 & \lambda^2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.18}$$

B.3.3 Sticky expectations

(Mankiw and Reis, 2002, Carroll, Crawley, Slacalek, Tokuoka and White, 2018). A date-0 shock ϵ causes a sequence of disturbances $\{dX_t\}$. At each date $t \geq 0$, some agents learn about ϵ and deduce $\{dX_{t+h}\}$ for all $h \geq 0$. The probability of learning ϵ is $1 - \lambda$ for every agent who hasn't learned it already. Thus the share of ignorant agents at date t is λ^{t+1}. They believe that the disturbances observed so far were special events, and don't expect any disturbances in the future. This setup implies that average expectations are $E_t[dX_{t+h}] = (1 - \lambda^{t+1}) \cdot dX_{t+h}$.

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1-\lambda & 0 & 0 & \dots \\ 0 & 0 & 1-\lambda & 0 & \dots \\ 0 & 0 & 0 & 1-\lambda & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 1-\lambda^2 & 0 & \dots \\ 0 & 0 & 0 & 1-\lambda^2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.19}$$

B.3.4 Noisy information and rational expectations

(Angeletos and Huo, 2021). A date-0 shock ϵ causes a sequence of disturbances $\{dX_t\}$ according to an MA process

$$dX_t = M_t \epsilon \tag{B.20}$$

Suppose that agents know the MA coefficients, M_t, but they don't observe ϵ. Their prior is that ϵ is distributed $\mathcal{N}(0, 1/\tau_\epsilon)$. At each date $t \geq 0$, agents receive independent private signals $\epsilon + \nu_t$,

where $\nu_t \sim \mathcal{N}(0, 1/\tau_\nu)$. Bayesian updating implies that the average posterior belief is

$$E_t[\epsilon] = \frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1}\epsilon \tag{B.21}$$

Then, the average expectation of dX_{t+h} at date t is

$$E_t[dX_{t+h}] = M_{t+h}E_t[\epsilon] = M_{t+h}\underbrace{\left(\frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1}\right)}_{\lambda_t}\epsilon = \lambda_t dX_{t+h} \tag{B.22}$$

Thus the associated Λ_t matrices are

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & \lambda_0 & 0 & 0 & \dots \\ 0 & 0 & \lambda_0 & 0 & \dots \\ 0 & 0 & 0 & \lambda_0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & \lambda_1 & 0 & \dots \\ 0 & 0 & 0 & \lambda_1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.23}$$

B.3.5 Extrapolation

Geometric extrapolation. $E_t[dX_{t+h}] = \lambda^h dX_t$. First example of non-diagonal Λ matrices.

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ \lambda & 0 & 0 & 0 & \dots \\ \lambda^2 & 0 & 0 & 0 & \dots \\ \lambda^3 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & \lambda & 0 & 0 & \dots \\ 0 & \lambda^2 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.24}$$

B.3.6 Adaptive expectations

(Cagan, 1956, Friedman, 1957). $E_t[dX_{t+h}] = \lambda^h \kappa \sum_{\tau=0}^{\infty} \lambda^\tau dX_{t-\tau}$, where $\kappa > 0$ scales the geometric sum.

$$\Lambda_0 = \kappa \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ \lambda & 0 & 0 & 0 & \dots \\ \lambda^2 & 0 & 0 & 0 & \dots \\ \lambda^3 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \Lambda_1 = \kappa \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ \lambda^2 & \lambda & 0 & 0 & \dots \\ \lambda^3 & \lambda^2 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.25}$$

B.3.7 Diagnostic expectations

(Bordalo, Gennaioli and Shleifer, 2018, Bianchi, Ilut and Saijo, 2021). Let $E_t^r[dX_{t+h}]$ denote a reference expectation for the variable h periods ahead. Then, the diagnostic expectation with parameter

θ is given by:

$$E_t[dX_{t+h}] = \mathbb{E}_t[dX_{t+h}] + \theta \left(\mathbb{E}_t[dX_{t+h}] - E_t^r[dX_{t+h}] \right).$$

Bordalo et al. (2018) assume that $E_t^r[dX_{t+h}] = \mathbb{E}_{t-1}[dX_{t+h}]$. In this case,

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 1+\theta & 0 & 0 & \cdots \\ 0 & 0 & 1+\theta & 0 & \cdots \\ 0 & 0 & 0 & 1+\theta & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad \Lambda_t = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.26}$$

for $t \geq 1$.

Bianchi et al. (2021) develop a generalization of this framework to allow for long memory, which assumes that $E_t^r[dX_{t+h}] = \sum_{j=1}^\infty \alpha_j \mathbb{E}_{t-j}[dX_{t+h}]$. With this assumption, we find

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 1+\theta & 0 & 0 & \cdots \\ 0 & 0 & 1+\theta & 0 & \cdots \\ 0 & 0 & 0 & 1+\theta & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 1+\theta(1-\alpha_1) & 0 & \cdots \\ 0 & 0 & 0 & 1+\theta(1-\alpha_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \tag{B.27}$$

and, for any t,

$$\Lambda_t = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 1+\theta(1-\sum_{j=1}^t \alpha_j) & 0 & \cdots \\ 0 & 0 & 0 & 1+\theta(1-\sum_{j=1}^t \alpha_j) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \tag{B.28}$$

B.3.8 Noisy information and diagnostic expectations

(Bordalo, Gennaioli, Ma and Shleifer, 2020) As in noisy information and rational expectations, the agent observes a signal $\epsilon + v_t$, where $v_t \sim \mathcal{N}(0, 1/\tau_v)$. However, the forecaster then overweighs representative states by using the distorted posterior

$$f^\theta(\epsilon|S_t^i) = f(\epsilon|S_t^i) R_t^i(\epsilon)^\theta \frac{1}{Z_t} \tag{B.29}$$

Bordalo et al. (2020) assume that $R_t^i(\epsilon) = f(\epsilon|S_t^i)/f(\epsilon|S_{t-1}^i \cup \{\mathbb{E}_{i,t-1}[\epsilon]\})$. This assumption implies that the mean of the distorted posterior is given by:

$$E_{i,t}^\theta[\epsilon] = \mathbb{E}_{i,t}[\epsilon] + \theta \left(\mathbb{E}_{i,t}[\epsilon] - \mathbb{E}_{i,t-1}[\epsilon] \right) \tag{B.30}$$

where $\mathbb{E}_{i,t}[\epsilon]$ denotes the time-t rational expectation with information set S_t^i. It follows that the average expectation is given by

$$\overline{E}_t^\theta[\epsilon] = \underbrace{\left[\frac{\left(\frac{t+1+\theta}{t+1}\right)\tau_\epsilon/\tau_\nu + t}{\tau_\epsilon/\tau_\nu + t}\right]\frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1}}_{\equiv \lambda_t}\epsilon \tag{B.31}$$

Thus the associated Λ_t matrices are

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & \lambda_0 & 0 & 0 & \cdots \\ 0 & 0 & \lambda_0 & 0 & \cdots \\ 0 & 0 & 0 & \lambda_0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & \lambda_1 & 0 & \cdots \\ 0 & 0 & 0 & \lambda_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \tag{B.32}$$

Analogously to Bianchi et al. (2021), we can extend this model to include long-memory as follows. Assume that $R_t^\theta(\epsilon) = f(\epsilon|S_t^i)/f^*(\epsilon|S_t^i)$, where $\epsilon \sim_{f^*|S_t^i} \mathcal{N}(E_t^r[\epsilon], \tau_\epsilon + (t+1)\tau_\nu)$ and $E_t^r[\epsilon] = \sum_{j=1}^t \alpha_j \mathbb{E}_{i,t-j}[\epsilon]$. It follows that

$$E_{i,t}^\theta[\epsilon] = \mathbb{E}_{i,t}[\epsilon] + \theta\left(\mathbb{E}_{i,t}[\epsilon] - E_{i,t}^r[\epsilon]\right). \tag{B.33}$$

As a result, the average expectation is given by

$$\overline{E}_t^\theta[\epsilon] = \left[(1+\theta)\frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1} - \theta\sum_{j=1}^t \alpha_j\left(\frac{t+1-j}{\tau_\epsilon/\tau_\nu + t + 1 - j}\right)\right]\epsilon, \tag{B.34}$$

and defining now $\lambda_t \equiv (1+\theta)\frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1} - \theta\sum_{j=1}^t \alpha_j\left(\frac{t+1-j}{\tau_\epsilon/\tau_\nu + t + 1 - j}\right)$ we obtain the analogous Λ_t matrices as above.

C Appendix to section 4

C.1 Financial intermediary

Set up decision problem formally and derive no-arbitrage conditions.

C.2 Retailers

The Bellman equation of firm j is

$$J_t(p_{jt-1}) = \max_{k_{jt}, l_{jt}, y_{jt}, p_{jt}} \left\{ \frac{p_{jt}}{P_t} y_{jt} - h_t l_{jt} - r_t^K k_{jt} - \frac{\psi_p}{2} \left[\log \left(\frac{p_{jt}}{p_{jt-1}} \right) \right]^2 Y_t + \frac{J_{t+1}(p_{jt})}{1 + r_t^e} \right\}$$

$$\text{s.t. } y_{jt} = F_t(k_{jt}, l_{jt})$$

$$y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\epsilon} Y_t$$

Substitute the production function and write the problem as

$$J_t(p_{jt-1}) = \max_{k_{jt}, l_{jt}, p_{jt}} \left\{ \frac{p_{jt}}{P_t} F_t(k_{jt}, l_{jt}) - h_t l_{jt} - r_t^K k_{jt} - \frac{\psi_p}{2} \left[\log \left(\frac{p_{jt}}{p_{jt-1}} \right) \right]^2 Y_t + \frac{J_{t+1}(k_{jt}, p_{jt})}{1 + r_t^e} \right\}$$

$$\text{s.t. } \frac{p_{jt}}{P_t} Y_t = \left(\frac{F_t(k_{jt}, l_{jt})}{Y_t} \right)^{-\frac{1}{\epsilon}} Y_t$$

Let η_{jt} denote the Lagrange multiplier on the constraint. The FOCs with respect to p_{jt} and p_{jt-1} are

$$0 = \frac{1}{P_t} F_t(k_{jt}, l_{jt}) - \psi_p \log \left(\frac{p_{jt}}{p_{jt-1}} \right) \frac{Y_t}{p_{jt}} - \eta_{jt} \frac{Y_t}{p_{jt}} + \frac{\partial_p J_{t+1}(k_{jt}, p_{jt})}{1 + r_t^e} \tag{C.1}$$

$$\partial_p J_t(k_{jt-1}, p_{jt-1}) = \psi_p \log \left(\frac{p_{jt}}{p_{jt-1}} \right) \frac{Y_t}{p_{jt-1}} \tag{C.2}$$

In symmetric equilibrium, the FOCs simplify to

$$0 = \frac{1}{P_t} F(u_t k_{t-1}, L_t) - \psi_p \log \left(\frac{P_t}{P_{t-1}} \right) \frac{Y_t}{P_t} - \eta_t \frac{Y_t}{P_t} + \frac{1}{1 + r_t^e} \psi_p \log \left(\frac{P_{t+1}}{P_t} \right) \frac{Y_{t+1}}{P_t} \tag{C.3}$$

$$0 = Y_t - \psi_p \log \left(\frac{P_t}{P_{t-1}} \right) Y_t - \eta_t Y_t + \frac{1}{1 + r_t^e} \psi_p \log \left(\frac{P_{t+1}}{P_t} \right) Y_{t+1} \tag{C.4}$$

$$\log (1 + \pi_t) = \frac{1}{\psi_p} (1 - \eta_t) + \frac{1}{1 + r_t^e} \frac{Y_{t+1}}{Y_t} \log (1 + \pi_{t+1}) \tag{C.5}$$

where $\pi_t = P_t / P_{t-1} - 1$ is the inflation rate. Define the real marginal cost as $mc_t \equiv (\epsilon - \eta_t)/\epsilon$. Then the equilibrium conditions can be summarized as

- Phillips curve:

$$\log (1 + \pi_t) = \frac{\psi_p}{\epsilon} \left(mc_t - \frac{\epsilon - 1}{\epsilon} \right) + \frac{1}{1 + r_t^e} \frac{Y_{t+1}}{Y_t} \log (1 + \pi_{t+1}) \tag{C.6}$$

- Labor demand:

$$h_t = mc_t \cdot \partial F_L(\tilde{K}_t, L_t) = mc_t(1 - \alpha) \frac{Y_t}{L_t} \tag{C.7}$$

- Capital demand:

$$r_t^K = mc_t \cdot \partial F_K(\tilde{K}_t, L_t) = mc_t \alpha \frac{Y_t}{\tilde{K}_t} \tag{C.8}$$

- Production:

$$Y_t = F_t(\tilde{K}_t, L_t) = \Theta_t \tilde{K}_t^\alpha L_t^{1-\alpha} \tag{C.9}$$

- Price adjustment cost:

$$\Psi_t = \frac{\psi_p}{2} \Big[\log\left(1 + \pi_t\right) \Big]^2 Y_t \tag{C.10}$$

- Dividends:

$$d_t^R = Y_t - h_t L_t - r_t^K \tilde{K}_t - \Psi_t \tag{C.11}$$

C.3 Capital producer

The Bellman equation is

$$J_t(K_{t-1}, I_{t-1}) = \max_{K_t, I_t} \left\{ r_t^K K_{t-1} - I_t + \frac{J_{t+1}(K_t, I_t)}{1 + r_t} \right\}$$
$$\text{s.t. } K_t = (1 - \delta)K_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] I_t \tag{C.12}$$

Let's define Tobin's Q as the marginal value of capital at the end of period t

$$Q_t \equiv \frac{\partial_K J_{t+1}(K_t, I_t)}{1 + r_t} \tag{C.13}$$

The FOC with respect to K_{t-1} is

$$\partial_K J_t(K_{t-1}, I_{t-1}) = r_t^K + \frac{\partial_K J_{t+1}^K(K_t, I_t)}{1 + r_t}(1 - \delta) \tag{C.14}$$

$$Q_t(1 + r_t) = r_{t+1}^K + Q_{t+1}(1 - \delta) \tag{C.15}$$

The FOC with respect to I_{t-1} is

$$\partial_I J_t(K_{t-1}, I_{t-1}) = \mu_t Q_t \left(\frac{I_t}{I_{t-1}}\right)^2 S'\left(\frac{I_t}{I_{t-1}}\right) \tag{C.16}$$

The FOC with respect to I_t is

$$0 = -1 + Q_t \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \left(\frac{I_t}{I_{t-1}}\right) S'\left(\frac{I_t}{I_{t-1}}\right) \right] + \frac{\partial_I J_{t+1}}{1 + r_t^e} \tag{C.17}$$

To summarize, the equilibrium conditions of the capital producer are

- Valuation:

$$1 + r_t = \frac{r^K_{t+1} + Q_{t+1}(1 - \delta)}{Q_t} \tag{C.18}$$

- Investment:

$$1 = Q_t \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \left(\frac{I_t}{I_{t-1}}\right) S'\left(\frac{I_t}{I_{t-1}}\right) \right] + \frac{\mu_{t+1} Q_{t+1}}{1 + r^e_t} \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right) \tag{C.19}$$

- Capital law of motion:

$$K_t = (1 - \delta)K_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] I_t \tag{C.20}$$

- Dividends:

$$d^K_t = r^K_t K_{t-1} - I_t \tag{C.21}$$

For concreteness, let the $S(\bullet)$ be quadratic

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{C.22}$$

$$S'\left(\frac{I_t}{I_{t-1}}\right) = \psi\left(\frac{I_t}{I_{t-1}} - 1\right) \tag{C.23}$$

C.4 Labor agency

The Bellman equation is

$$J_t(N_{t-1}) = \max_{N_t, v_t} \left\{ (h_t - w_t)N_t - (\kappa_v + \kappa_h q_t)v_t + \frac{J_{t+1}(N_t)}{1 + r_t} \right\} \tag{C.24}$$

$$\text{s.t. } N_t = (1 - s_t)N_{t-1} + q_t v_t$$

Let λ_t denote the Lagrange multiplier on the constraint. The FOCs wrt N_t, v_t, and N_{t-1} are

$$0 = h_t - w_t - \lambda_t + \frac{J'_{t+1}(N_t)}{1 + r_t} \tag{C.25}$$

$$0 = -\kappa_v - \kappa_h q_t + \lambda_t q_t \tag{C.26}$$

$$J'_t(N_{t-1}) = \lambda_t(1 - s_t) \tag{C.27}$$

Combining these yields the job creation curve. In sum, the equilibrium conditions are

- Job creation:

$$\frac{\kappa_v}{q_t} + \kappa_h = h_t - w_t + \frac{1 - s_{t+1}}{1 + r_t}\left(\frac{\kappa}{q_{t+1}} + \kappa_h\right) \tag{C.28}$$

- Dividends:

$$d_t^L = (h_t - w_t)N_t - (\kappa_v + \kappa_h q_t)v_t \tag{C.29}$$

C.5 Household expectations

The aggregate variables that households must forecast to make their consumption-saving decision are $\{f_t, \pi_t^{lose}, w_t, r_t^a, d_t^{FI}\}$. We have expectations data on the unemployment rate, which is closely related to the job-finding rate f_t, and the UI expiration probability π_t^{lose}. We construct expectations for $\{f_t, \pi_t^{lose}\}$ by assuming that households have first-order knowledge of the structural relationships between these variables and the unemployment rate.

In matrix notation, we look for Jacobians $\mathcal{J}^{f,U}$ and $\mathcal{J}^{\pi^{lose},U}$ such that

$$d\mathbf{f}^{e,t} = \mathcal{J}^{f,U} d\mathbf{U}^{e,t} \qquad \text{and} \qquad d(\boldsymbol{\pi_t^{lose}})^{e,t} = \mathcal{J}^{\pi^{lose},U} d\mathbf{U}^{e,t} \tag{C.30}$$

First, the relationship between the unemployment rate and UI expiration rate is given by the policy rule

$$\frac{1}{\pi_t^{lose}} - \frac{1}{\pi_{ss}^{lose}} = \zeta_b (U_t - U_{ss}) \tag{C.31}$$

Differentiating the policy rule at the steady state yields

$$d\pi_t^{lose} = - \left(\pi_{ss}^{lose}\right)^2 \zeta_b dU_t \tag{C.32}$$

which shows that $\mathcal{J}^{\pi^{lose},U}$ is diagonal matrix.

Second, the relationship between the employment rate and job-finding rate is given by the law of motion

$$U_t = (1 - f_t)U_{t-1} + s(1 - f_t)(1 - U_{t-1}) \tag{C.33}$$

$$= (1 - f)(1 - s)U_{t-1} + s(1 - f_t) \tag{C.34}$$

Differentiating this law of motion yields

$$df_t = \frac{(1 - f)(1 - s)dU_{t-1} - dU_t}{s + (1 - s)U} \tag{C.35}$$

which shows that $\mathcal{J}^{f,U}$ is a lower bidiagonal matrix.

D Appendix to Section 5

D.1 Fitting a model of beliefs

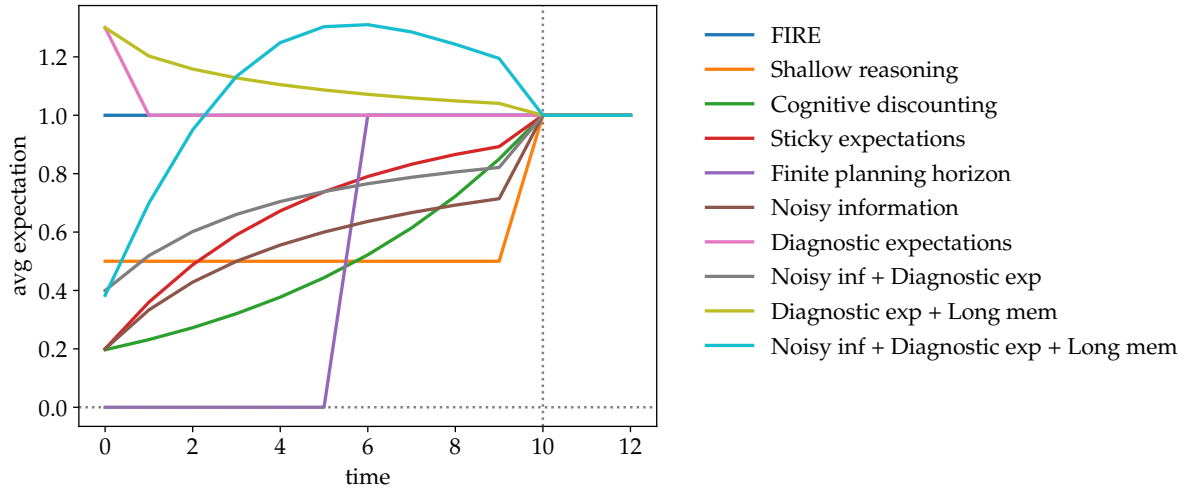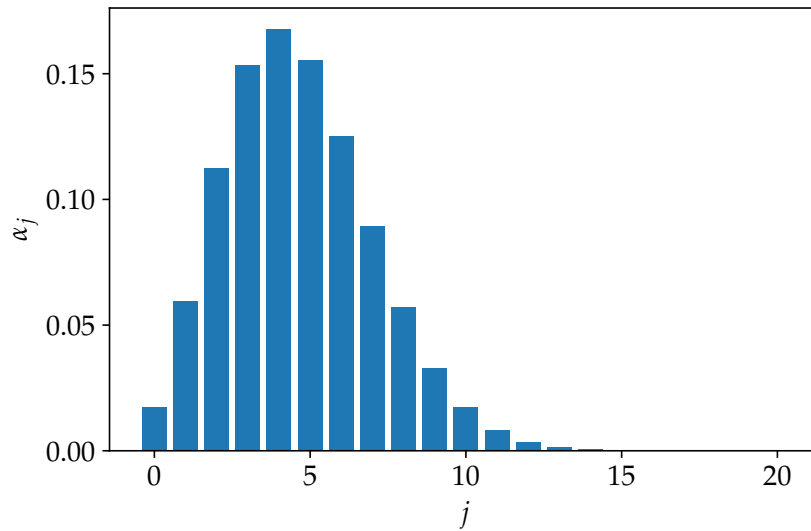Figure D.1: Illustration of parametric belief models

Figure D.2: Estimated memory weights

D.2 Decomposing GE forces

Figure D.3 shows the decomposition of the aggregate consumption response into all the variables that enter the aggregate consumption function directly for each model of belief formation used in

our analysis. The job-finding rate and UI duration emerge as the main drivers of consumption response in the first four quarters. The shape of the consumption response to the UI duration is recognizable from the partial equilibrium exercise.

Figure D.3: Decomposition of general equilibrium consumption response